Platonic Ontology and Stereometry:

Evidence from the *Philebus* and the *Laws*

Chiye Izumi

I deal first with the *Philebus*, which introduces the four-fold classification of reality, namely, limit, unlimited, generation into reality, and causes. It is this classification which plays a crucial role in Plato's later ontology where intelligibility depends heavily upon the role of limit. Limit in turn is glossed in terms of number and measure or, rather more importantly, ratios within them. *Ratio* theory (the theory of proportion) is generally thought to have developed from a notion that is applicable only to commensurable magnitudes to one that encompasses ratios and proportions also between incommensurable ones.[1] So insofar as limit is exemplified by ratios (between number and number, or measure and measure) that can go beyond the commensurable cases and include some incommensurables, viz. the ratios between them. Although the *Philebus* does not explicitly mention stereometry, the notion of limit in the *Philebus* has important implications for the topic of incommensurability, which is one of the subjects to which stereometry is applied in Plato's later thought.

Second, I shall deal with the *Laws*, where the classification of μαθήματα is very different from that of the *Republic*. The *Laws* does not seem to presuppose the sequence of mathematical objects (numbers, planes, solids and moving solids and moving numbers) which the *Republic* presents when it introduces stereometry. Moreover, the *Laws* does not regard stereometry as an independent mavqhma as the *Republic* does. Stereometry is included in the art of measurement (μετρητική)which is defined as the subject dealing with 'line, plane and depth', and the art of measurement is explicitly related to the study of incommensurability (*Laws* 817e ff). I shall compare the

[1] See, for example, Heath 1921: 153, 155, 167, 216, 325-7.

configuration of μαθήματα of the *Laws* to that of the *Republic*, and discuss the relation between the art of measurement and stereometry. This will help to illustrate further aspects of stereometry's contribution to the resolution of problems connected with incommensurability. Moreover, I shall use a study of the relation between the art of measurement and the study of numbers, to investigate where the configuration of μαθήματα as it appears in the *Laws* diverges from and also where it develops earlier ideas.

1. The *Philebus*: the fourfold classification of reality and the new framework of mathematical studies and its emphasis on commensurability

The *Philebus* shows us a certain development of the Timaean motif that 'the Demiurge creates the universe out of numbers and shapes', and the development seems to correlate with the problem of incommensurability which is not fully discussed in the *Timaeus*. The fourfold classification of reality which Plato introduced in the *Philebus* (limit, unlimited, generation into reality and causes) seems to reflect not merely the so-called Pythagorean doctrine of Limit and Unlimited which is here referred to as 'heavenly tradition', but also Plato's insight into *ratio* theory which is being applied to incommensurable quantities. The following passage which describes the class of the limit deserves attention:

> Socrates: 'The things which do not admit of more and less and the like, but do admit of all that is opposed to them - first equality and the equal, then the double, and all that is related as number to number or measure to measure - all these might properly be assigned to the class of the limit. Do you agree ?'
> Protarchus: 'Completely.' (*Philebus* 25b)

It seems important to note that Plato does not describe the class of the limit in terms of merely numbers and shapes, -- which, we should recall, are the key terms in the Timaean motif that 'the Demiurge creates the universe out of numbers and shapes'.[2] While numbers and shapes play in the *Timaeus* the crucial role for giving orderliness to the chaotic state of the universe, they do not play the same role in the *Philebus*. Instead of 'numbers and shapes', the *Philebus* emphasises the *relation* between 'number to number' and 'measure to measure'. Moreover, it should be noted that the term 'shapes' does not appear in the above passage but instead the word 'measure' does. The replacement of 'shapes' by 'measures' and also the emphasis on the concept 'relation' between 'number to number' and 'measure and measure' can be construed as an indication that what may lie behind this passage is the theory of proportion, particularly in its application to the case of incommensurables. Knorr reads the above passage in the light of the problems concerning the measurement of incommensurable quantities. He says: 'Not all incommensurable lines are expressible as rational combination of 1 and square root of 2... In the incommensurable case, the measures themselves will be incommensurable. Plato also seems to refer to such a notion, when in the *Philebus* (25a ff) he speaks of the relation of "number to number and measure to measure" '.[3] Moreover, as for the possibility that the problems concerning the measurement of incommensurable quantities might have lain behind the fourfold classification of reality in the *Philebus*, we may offer the following points. The Timaean key terms, viz., numbers and shapes, seem too weak to deal with the problems of incommensurable quantities. Likewise, the ontological sequence of mathematical objects (numbers, planes, solids, the movement of solids and the movement of numbers) which we find in the *Republic* does not seem useful for resolving the problems of incommensurability. Only the fourfold classification of reality as it appears in the *Philebus* can provide the ontological rationale for dealing with the problems of incommensurable quantities.

The class of the limit gives measures to things which belong to the class of the unlimited. While most of Plato's examples are physical ones (hotter, colder, disease, the

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[2] *Timaeus* 53b.

[3] Knorr, 1975: 205-6, n.27.

weather, musical sounds),[4] we are to understand that the way in which limit is exemplified is via the mathematical concepts of number and measure, and the effect of their introduction is to make the 'mixed' class not just 'harmonious' (σύμφωνα, 25e1), but precisely, commensurable (συμμετρία, 25e1, cf. 26a8, 65a2, 66b1, ἔμμετρον 26a7). We should not fail to remark that the addition of the more generic term 'mevtron' to the more specifically arithmetic ' ἀριθμός ' at 25a8-b1 allows coverage of commensurables of all types, including geometrical ones and ones dealing with other continua such as that of musical sound, over and above those συμμετρία that may be expressed as ratios between integers. Thus, even incommensurable ratios can be understood by means of the notion of proportion.[5] This may be a crucial background to the ontology and epistemology set out in the *Philebus*. In the ending of the *Philebus* (66a ff.), where Plato divides what he calls 'possessions (κτῆμα)' of mankind into five, on a scale of decreasing value, we find clearly that the concept 'μέτρον' wins the first prize; 'proportion (σύμμετρον)' comes second, and 'intelligence (νοῦς)' comes third. Although this order does not satisfy scholars believing the primacy of ethical values over mathematical ideas,[6] we might suggest that the ordering clearly reflects the essence of the limit as we saw above.

The emphasis on the relation of 'number to number' and 'measure to measure' seems to influence the classification of mathematical subjects later in the *Philebus*. Although it is well-known that in the *Philebus* Plato tries to divide mathematical subjects into two types, namely broadly empirical ones and theoretical ones, I would suggest, rather, the following remarks at *Philebus* 55e also demand attention here: 'If someone were to take away arithmetic, the art of measurement, and weighing from the arts and crafts, the rest might be said to be worthless'. Here it seems that the *Philebus* does not suppose the *Republic* -type framework of the five mathematical subjects (the

4 *Philebus* 25e ff.

5 See, Heath 1921: vol. 1, 326-327; Euclid V, Def. 5.

6 See, for example, Guthrie 1978: vol. 5, 235. Together with ' μέτρον ', ' καιρόν ' and 'everything that must be thought to be of this sort' come first. The second includes 'suvmmetron', beauty, perfection, sufficiency, and all that belongs to that class. The third includes intelligence and ' φρόνησις '.

study of numbers, geometry, stereometry, astronomy and harmonics). When mathematical subjects are classified at *Philebus* 55e, the first two mentioned are the study of numbers and the art of measurement. Thus far this primary twofold division corresponds to the two kinds of mathematical entities referred to in the description of the class of the limit at 25a-b, namely numbers and measure. Indeed that these are the primary mathematical subjects is confirmed when, at 57d it is said that true philosophers are immeasurably superior in accuracy and truth about measures and numbers. There are two arts of arithmetic and two arts of measuring, divided according to their degree of accuracy and abstraction.

Thus, concerning the framework of mathematical studies, the *Philebus* contains elements which neither the *Republic* nor the *Timaeus* suggests. The emphasis on 'numbers and measures' and also on the art of measurement is not found in those two works.[7] Those emphases can be taken as an indication of the tendency towards a new view of mathematical subjects found in both the *Laws* and the *Epinomis*.

2. The *Laws*

Here the *Laws* must be explored: stereometry appears in *Laws* VII as a part of the art of measurement dealing with commensurable/incommensurable relations between lines, planes and solids; moreover, the *Laws* emphasises the concepts 'number and measure', which are also emphasised in the *Philebus*, not only in worldly problems but also in the cosmic perspective.

7 However, we find in *Republic* X that the importance of 'measurement' and 'counting' is emphasised. See, *Republic* X 602d-603a. As regards the significance of 'measurement' as it appears in *Republic* X, see, G.E.R.Lloyd 1987: 241 n. 100, 299 n. 52.

2.1 *Laws* VII 819d-820c: what is indicated by commensurable / incommensurable relations between 'line', 'plane' and 'depth' ?

When Plato introduces in *Laws* VII three mathematical studies for free-born children (the three mathematical studies being the study of numbers, the art of measurement and astronomy), he abruptly refers to the 'shameful Greek ignorance' of what is commensurable or not (*Laws* VII, 819d-e). His claim here is that this ignorance can be dispelled by the art of measurement which clarifies commensurable/incommensurable between 'line', 'plane' and 'depth'. Although stereometry does not explicitly appear in the list of the three mathematical studies in *Laws* VII, it seems possible to take the art of measurement to include stereometry, that is as well as geometry and whatever other modes of measurement may be in Plato's mind.

It is possible, then, that *Laws* VII can provide us with clues concerning the importance of stereometry for problems of incommensurability in Plato's later thought. I shall explore the passage from *Laws* VII 817e to 820e, focussing on the following four points: (1) what is indicated by commensurable / incommensurable relations between 'line', 'plane' and 'depth'; (2) the relation between stereometry and the art of measurement; (3) the relation between the art of measurement and the study of numbers.

The description of the art of measurement in *Laws* VII begins with the lamentation of 'our [Greek] ignorance' of the measuring of line, plane and depth (819d). The Athenian Stranger who came to perceive the ignorance rather belatedly (ὀψέ ποτε) was utterly astounded and compared the ignorance even to 'the condition of guzzling swine rather than of human beings'(819d). He says, 'I blushed not for myself alone, but for our whole Hellenic world'(819e). Although it seems somewhat exaggerated and even humorous, Athenian Stranger's lamentation of the Greek ignorance might indicate Plato's keen interest in, and preoccupation with problems relating to the art of measurement of line, plane and depth. However, the substance of the ignorance is not clear. Cleinias, the interlocutor of the Athenian Stranger, asks him to explain the ignorance (819e). Then, the Athenian Stranger begins to do so by asking him questions in return.

> Athenian Stranger: 'Pray tell me one little thing. You know what is meant by line?'
> Cleinias: 'Of course I do.'
> Athenian Stranger: 'And by surface ?'
> Cleinias: 'Certainly.'
> Athenian Stranger: 'And do you know that they are two distinct things, and that the third thing, next to these, is depth ?'
> Cleinias: 'I do'.
> Athenian Stranger: 'Do you not, then, believe that all these are commensurable one with another ?'
> Cleinias: 'Yes.' (*Laws* VII, 819e)

Although Cleinias thus answers 'Yes' to the leading question of whether *all* (line, plane and depth) are commensurable one with another (πάντα μετρητὰ πρὸς ἄλληλα), the problem centres on his agreement 'all'. But the correction of Cleinias by the Athenian stranger shows that Cleinias is represented as not realising there are any instances of incommensurables at all. The Athenian Stranger, then, continues to ask Cleinias the following cases.

> (1) Line is in its very nature commensurable with line, surface with surface, depth with depth. (Μῆκός τε οἶμαι πρὸς μῆκος, καὶ πλάτος πρὸς πλάτος, καὶ βάθος ὡσαύτως δυνατὸν εἶναι μετρεῖν φύσει..) (819e14-820a2)
> (2) Some of them are neither with more assurance, nor with less, commensurable, some being commensurable and some not. Εἰ δ' ἔστι μήτε σφόδρα μήτε ἠρέμα δυνατὰ ἔνια, ἀλλὰ τὰ μέν, τὰ δὲ μή.) (820a4-5)
> (3) As regards the relation of line and surface to depth, or of surface and line to each other, these might be somehow commensurable with one another. (Μῆκός

τε οἶμαι πρὸς μῆκος, καὶ πλάτος πρὸς πλάτος, καὶ βάθος ὡσαύτως δυνατὸν εἶναι μετρεῖν φύσει.) (820a8-12)

I shall now re-assess statement (1). Cleinias in *Laws* VII regards (1) as 'absolutely correct' while the Athenian Stranger avoids making a judgement of whether or not Cleinias is right. The least contentious interpretation of (1) may be that each of length, area and volume has its own unit of measurement. As Eva Sachs points out, the interpretation could be endorsed by some ancient episodes suggesting that the Greeks may have made mistakes in dealing with standardised units of measurement.[8] According to Thucydides (6.1.2), most of the Athenians were ignorant of the size of Sicily, even though they knew that it took not much less than eight days for the voyage round the island; this episode is sometimes taken to suggest that the Greeks made a mistake to measure areas by units of lengths. Likewise, Quintilian refers to a similar episode that the Greeks regarded the time taken to circumnavigate an island as a sufficient indication of its size (1.10.40). If we connect these episodes with statement (1), we can take the statement to indicate the mathematical truth which the Greeks should have known concerning the units of the measurements, namely that the line should be measured by the unit of length, the surface by that of area, the volume by that of volume.

However, another interpretation of statement (1) is also possible, if we take problems of incommensurability into consideration. Certain lines are incommensurable with the unit of length; certain surfaces incommensurable with the unit of area; (certain volumes incommensurable with the unit of volume). When such incommensurable quantities are observed, it is not true to say that line is measurable with line, surface with surface, depth (volume) with depth (volume). Then, statement (1) becomes false, although it appears to Cleinias absolutely true.

This line of interpretation of statement (1) is endorsed by statement (2), that is, 'some of them (line, surface and volume) are neither with more assurance, nor with less, commensurable, some being commensurable and some not (ἔστι μήτε σφόδρα μήτε ἠρέμα δυνατὰ ἔνια, ἀλλὰ τὰ μέν, τὰ δὲ μή)'(820a4-5). I would take this statement,

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8 Eva Sachs 1917: 174 ff.

though being opaque, to indicate that there are lines (areas, volumes) σφόδρα commensurable with lines (areas, volumes), while there are lines hjrevma commensurable with lines but commensurable in square. The unit of length (areas, volumes) is not always valid in the measurements of length (areas, volumes). In this sense, statement (2), which gives a counter-example to statement (1), can be regarded as true. (We might also notice that the term sfovdra is an echo of Cleinias' over emphatic answer ' Σφόδρα γε '.)

Thus, the shift from statement (1) to statement (2) reveals a new dimension, where the incommensurability between lines (areas) is to be observed. However, if a volume is commensurable with another volume, lines and planes constituting the volumes are not necessarily commensurable with one another. Statement (3) says, 'as regards the relation of line and surface to depth, or of surface and line to each other, these might be, in some way, commensurable with one another (μῆκός τε καὶ πλάτος πρὸς βάθος, ἢ πλάτος τε καὶ μῆκος πρὸς ἄλληλα; [ὥστε πῶς] ἆρ' οὐ διανοούμεθα περὶ ταῦτα οὕτως Ἕλληνες πάντες, ὡς δυνατά ἐστι μετρεῖσθαι πρὸς ἄλληλα ἁμῶς γέ πως;.)' (820a8-12).

Concerning statement (3), the Athenian Stranger adds the following comment at 820b: 'if they (the relation of line and surface to volume, or surface and line to each other) cannot be measured *by any way or means* (μηδαμῶς μηδαμῇ), while, as I said, all we Greeks think that they can, are we not right in being ashamed for them all ?' This is usually interpreted as a further lamentation of Greek ignorance. However, it does not seem a straightforward lamentation. We need to pay attention to the expression 'if they *cannot* be measured *by any way or means*'. If we recall here the *Theaetetus* passage (147b1-148d) discussed above, we might realise that it is not always true that 'they *cannot* be measured *by any way or means*.' Even if a line is not commensurable with the unit of length, it can become commensurable by the areas (or volumes) it can form. Yet for the Greeks to have assume that all lines are commensurable is clearly a blunder.

Thus, while the discussion concerning the commensurability of 'line', 'plane' and 'depth' begins with the problems of the units of measurements, the problems seem to develop into a higher study of incommensurability. At 820c, the Athenian stranger explicitly refers to the study which deals with 'problems concerning the essential nature of the commensurable and the incommensurable (Τὰ τῶν μετρητῶν τε καὶ ἀμέτρων πρὸς ἄλληλα ἥτινι φύσει γέγονεν)'.

It should be noted that the description of the art of measurement in *Laws* VII thus ends with the reference to the study of 'commensurability and incommensurability of whatsoever type'. This seems important for our next attempt to elucidate the relation between the art of measurement and the study of numbers.

2.2 The relation between measurement (μετρητική) and arithmetic (ἀριθμητική)

The above depiction of art of measurement (μετρητική) in *Laws* VII brings us to the question which is of central importance for our next discussion about the place of stereometry in Plato's later period, the interpretation of the relation between art of measurement (μετρητική) and arithmetic (ἀριθμητική). We have seen, from our analysis of *Philebus* 55e-57a, that both metrhtikhv and ajriqmhtikhv may have a practical/ applied and a pure/ philosophical part. The question we shall address in this section is how stable and how clear is the distinction between μετρητική and ἀριθμητική. We shall find that the relationship between these two is construed differently in different late Platonic texts, and this clearly has a bearing both on the classification of the maqhvmata in general and on the place of stereometry in particular, as well as on Plato's views on the problems of commensurability and incommensurability.

There was a famous issue as to whether arithmetic, or geometry, is prior. The chief texts have been set out, but so far as Plato is concerned, *Republic* VII (526b), *Laws* VII (817e, 819b) are among the more important passages suggesting the primacy of arithmetic (cf. also *Epinomis* 977e to be discussed below), while the study of the general injunction of the need to study geometry to enter the Academy -- along with the importance of geometry in the *Theaetetus* and the *Meno* -- suggest that , at least from some points of view, as much attention is paid to geometry as to arithmetic.[9]

The evidence just considered suggests a struggle for primacy between arithmetic and geometry, with the boundary between them being clearly defined. The picture is brought into some doubts, however, when we ask whether the boundary between arithmetic (ἀριθμητική) and the art of measurement (μετρητική) is always itself clearly drawn.

There are occasions where the boundary between them seem less clear. For example, consider *Politicus* 284e:

> Eleatic Stranger: 'It's clear we would divide the art of measurement, cutting it in two in just the way we said, positing as one part of it all those kinds of expertise that measure the number, lengths, depths, breadths and speeds of things in relation to the opposite, and as the other, all those that measure in relation to what is in due measure, what is fitting, the right moment, what is as it ought to be -- everything that removes itself from the extremes to the middle'. (*Politicus* 284e2-8)[10]

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9 The famous imperative reported by ancient commentators to have been inscribed over the gates of Plato's Academy : 'Let no one who is ignorant of geometry enter here' (ΑΓΕΩΜΕΤΡΗΤΟΣ ΜΗΔΕΙΣ ΕΙΣΙΤΩ). cf. Ar. Gr. XV 117. 29; XVII 118.18. In the *Theaetetus* (145a, 145c) geometry is the subject best characterising the mathematical ability of Theodorus and Theaetetus. The importance of geometry is also related to the theory of recollection. *Meno* 82a-85b, 85e-d; *Phaedo* 73a. The view that geometry best characterises the Platonic education framework is also detected in Plato's *Epistle* III, where Dionysius II refers in particular to 'geometry' when he describes the education he received from Plato (319c). According to Plutarch, when Plato visited Dionysius II, 'the palace was filled with dust, owing to the multitude of geometricians there' (*Dion*, 13). Drawing geometrical figures on loose sand strewn upon the floor is the causes of the dust. In the *Quaestiones Convivales* (8. 2), Plutarch discusses the dictum ascribed to Plato, that God is always doing geometry, and illustrates Plato's philosophy in the light of the significance of geometry. Moreover, in Vitruvius (*De Arch*. 6.1) and Cicero (*Rep*. 1. 29), 'geometrical figures' appear as a symbol of 'the tracks of men (homimium vestigia)' and 'the indications of learning (doctrinae indiciis)'. Philolaus A7a and Archytas B4 are also relevant here.

10 I follow Rowe's translation. Rowe 1995.

Although the above passage speaks of two kinds of measurement, we might be surprised that the objects of the art of measurement include 'the number, lengths, depths, breadths and speeds'.[11] This means that arithmetic, geometry, stereometry and astronomy, which constitute the μαθήματα in *Republic* VII, are subsumed under the province of this art of measurement (μετρητική). The boundary between an art of measurement (μετρητική) and arithmetic (ἀριθμητική), then, here becomes blurred. As Young Socrates appearing in the *Politicus* says, this province of the art of measurement as it appears at *Politicus* 284e ff is 'indeed a vast section (μέγα τμῆμα)'. This view of the art of measurement involving other areas of μαθήματα might be ascribed to the 'κομψοί' who claim at *Politicus* 285a1-2 that 'the art of measurement relates to everything that comes into being'. Yet, as to the problem of who the 'κομψοί' were, there is no solid evidence. Campbell says that they were Pythagoreans while Rowe claims that they were '(Pythagoreanising ?) members of the Academy.[12] Regardless of who the 'κομψοί' might have been, what is more important for us is to consider why arithmetic (ἀριθμητική) came to be included in the province of μετρητική. The question seems relevant to the following passages where, on the contrary, the art of measurement is subsumed under the province of arithmetic:

> Eleatic Stranger: '[if we were to see] all the science of numbers [arithmetic], whether - I imagine - dealing with pure numbers, or plane, or in depths, or in speeds, - in relation to all of these things, practised in this way, what on earth would be the result that would appear, if they were done on the basis of written rules and not on the basis of expertise ?' (*Politicus* 299e2ff)

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11 It should be noted that ' τὸν ' attaches to 'number (ἀριθμὸν)' while 'lengths, depths, breadths and speeds' are plural nouns without articles. Professor G. Lloyd suggested that the passage can be translated as 'measuring number, and 'lengths, depths, breadths and speeds' in terms of number, taking ' τὸν ἀριθμὸν ' as accusative of respect. Moreover, it should be noted that, instead of 'ταχυτῆτας', 'παχυτῆτας' appears in some manuscripts (Bw).

12 Campbell 1897: 107; Rowe 1995: 209.

Apart from the context of the above passage which is concerned with which subjects may be studied and the limits of those subjects,[13] it seems prudent to pay attention to the point that arithmetic (ἀριθμητική) is described as the study which deals not merely with pure numbers but also with other numbers relating to 'plane', 'depths' and 'speeds'. The expression `σύμπασαν ἀριθμητικὴν ψιλὴν εἴτε ἐπίπεδον εἴτ' ἐν βάθεσιν εἴτ' ἐν τάχεσιν οὖσάν που '(299e1-9), though being somewhat opaque, can be taken to indicate certain 'number sets' constituted by 'pure numbers', 'numbers relating to plane', 'numbers in depth' and 'numbers in speeds'. What precisely may be in Plato's mind is unclear, but one possibility is that 'all arithmetic' (σύμπασα ἀριθμητικὴ) here embraces not just arithmetic but also certain aspects at least of what is elsewhere more generally described as belonging to the art of measurement. viz. when that deals with plane and solid geometry and speeds. Although the word 'που (perhaps)' suggests that such an assimilation is subject to qualification, we seem to have a contrast with the case at *Politicus* 284e2-8, where the province of the art of measurement (μετρητική) includes not only arithmetic (ἀριθμητική) but also other maqhvmata. Before investigating why arithmetic (ἀριθμητική) is to be assimilated to the art of measurement (μετρητική) or vice versa, we must look at the following passage in *Laws* V:

> The Athenian Stranger: 'He [the lawgiver] must recognise it as a universal rule that the divisions and variations of numbers are applicable to all purposes -- both to their own arithmetical variations and to the variations in terms of length and in depth, and also to those of sounds, and of motions, whether in a straight line up and down or circular.'
> (καὶ κοινῷ λόγῳ νομίσαντα πρὸς πάντα εἶναι χρησίμους τὰς τῶν ἀριθμῶν διανομὰς καὶ ποικίλσεις, ὅσα τε αὐτοὶ ἐν ἑαυτοῖς ποικίλλονται καὶ ὅσα ἐν μήκεσι καὶ ἐν βάθεσι

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[13] *Politicus* 299b ff.

> ποικίλματα, καὶ δὴ καὶ ἐν φθόγγοις καὶ κινήσεσι ταῖς τε κατὰ τὴν εὐθυπορίαν τῆς ἄνω καὶ κάτω φορᾶς καὶ τῆς κύκλῳ περιφορᾶς.) (*Laws* V, 746e-747a)

Here we see again that arithmetic (ἀριθμητική) embraces geometry, stereometry, astronomy and harmonics, these being the maqhvmata as they appear in *Republic* VII. This kind of unification of maqhvmata might reflect the view that the objects of maqhvmata are described in terms of 'the variations of numbers'.

We see then that two rather conflicting tendencies appear in different passages in late Platonic works. One view has it that arithmetic is a study that in some sense embraces all the other branches of mathematics. The other has it that the art of measurement is a generic discipline, subdivided into arithmetic and other areas.

Now Aristotle, in a passage in the *Metaphysics* (1020a7ff) that I have cited before (in chapter 3), arrives at a resolution of the problem by way of his introduction of the category of quantity (ποσόν) that embraces both plurality (πλῆθος) and magnitude (μέγεθος).[14] But while that gives a clear taxonomy, within which the different and distinct subject-matters of arithmetic on the one hand, geometry on the other, can each be given their place, Plato, for his part, has no such category of quantity as such. Reflecting, as we have seen, the earlier tension in the dispute on the primacy of arithmetic and geometry, Plato adopts different orderings of the chief mathematical disciplines in different contexts and for different purposes.

Very broadly speaking, the privileging of ἀριθμητική, corresponds to a view that number is in some sense prior to and simpler than line, plane and solid. In that context Plato does not just suggest that the study of mathematics should start with arithmetic before progressing to the study of mathematical objects with two and higher dimensions. In some passages (as we have seen in *Politicus* 299e) it seems that other mathematical studies are, in some sense, reducible to numbers. We shall see the full development of that idea when we come to tackle the *Epinomis*. But for that reduction to be carried

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14 See also Aristotle *Physics* 221b14 ff.

through, one or other or both of two ideas have to be exploited. The first is that numbers have to be treated as including shapes, the second that shapes have to be seen as corresponding in some sense to numbers. Both ideas have their precursors in what is reported of Pythagorean ideas, but both seem to offer possible lines of interpretation of Platonic texts.

Yet the stronger the link between other branches of mathematics and arithmetic, deemed to be prior to the rest, the greater the potential problem posed by incommensurabilities. To cope with incommensurables the Greeks turned naturally to geometry. The relationship between the side and the diagonal of square was, indeed, not a relationship between two numbers (one rational, the other irrational), because the notion of an irrational number was not normally countenanced. Moreover, as we have seen, the key procedure adopted to show how some incommensurables were, after all, commensurable, and so in good order, was to move from lines to planes, and from planes to solids. Geometry and stereometry there performed invaluable services in securing order where disorder threatened. It is that idea that is often in the background when metrhtikhv, rather than ajriqmhtikhv, is given primacy among the mathematical studies. It is by measuring things that the elements of limit and order in them can be discerned -- and that applies to what is numbered as much as what is measured in the fields of geometry, stereometry, astronomy and harmonics. As *Politicus* 284e shows Plato prefers measurement against the *due* measure to measurements of one thing against another. But his keenness on measurement of all kinds sometimes leads him to focus on it as the key issue for all the branches of mathematics to face.

That Plato himself perceives no conflict between these two groups of ideas, the privileging of ajriqmhtikhv, and that of metrhtikhv, is strongly suggested by the fact that *both* figure in a single dialogue, the *Politicus*. But that perhaps underlines that his interests were not solely in the abstract analysis of the ontology of mathematical entities, nor, again, solely in the links provided by the theme of measurement. He is, undoubtedly, deeply concerned with the notion of order and limit, with all of their applications, both mathematical and non-mathematical. But he allows his mathematical working out of those notions to vary in different contexts.

2.3 The significance of 'number' and 'measure' in the *Laws* : the mathematical foundation of divine necessity

I shall now present considerations suggesting that the concepts 'number' and 'measure', which we have so far been traced in the *Philebus* and the *Theaetetus*, play important roles in the *Laws* in underpinning not merely laws governing worldly and civic affairs but also the framework of the three μαθήματα (arithmetic, the art of measurement and astronomy) in *Laws* VII, and furthermore the vision of the universe which we may detect in the *Laws*.

However, it might be objected that the three maqhvmata in *Laws* VII simply constitute part of the elementary education of the ordinary people and do not carry any philosophical message. The criticism might also be made that what is presented as a view of the universe in the *Laws* is at most the theological view that 'souls control the movements of the universe', and that we must wait for the so-called thirteenth book of the *Laws*, the (possibly inauthentic)*Epinomis*, before we find a vision of the universe founded on 'number' and 'measure'. Indeed, since Zeller,[15] the majority of scholars have tended to deal with the *Laws* merely in the light of Plato's view of education or his system of laws. Jaeger, for example, claims: 'The *Laws* contains neither logic nor ontology. . . it contains most profound discussions of the state, of law, of morals, and of culture. But all these subjects Plato subordinates to paideia [education].'[16] A similar view was propounded also by Morrow. As Kahn puts it in his foreword, 'Morrow interprets the *Laws* primarily as a system of legislation rather than as a work of theoretical philosophy.'[17] Such views reflect, even though they may not be derived from, a recognition that 'the word φιλοσοφία does not appear in the *Laws*, and its cognates only rarely; and we hear little of the familiar doctrine of Ideas.'[18]

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15 See, Jaeger's brief survey of the history of the interpretations of the *Laws*. Jaeger 1944: vol. 3 213.

16 Jaeger *ibid*..

17 See, Kahn's foreword to Morrow (1993: xvii).

18 Morrow 1993: 573.

My interest in the *Laws*, however, lies neither in 'education' in the sense discussed by Jaeger nor in 'a system of legislation' as detailed by Morrow. Rather, I will explore in the *Laws* the view of the universe that is suggested by the framework of the μαθήματα that is there proposed.

A clue to the view of the universe implicit in the *Laws* is the word 'ἀναγκαῖα (necessity)'. In *Laws* VII at 818a-e, Plato seems deliberately to exploit the semantic range of the word 'ἀναγκαῖα (necessity)', in order to make a linkage between 'the indispensability of μαθήματα (αὐτῶν ἀναγκαῖα)' (818a4), 'mathematical necessity (τὸ ἀναγκαῖον αὐτῶν)' (818a7) and 'divine Necessity (θεῖαι τῶν ἀναγκῶν)' (818b3, 818b7-8). Such a linkage of the words 'necessity' might appear strange, because we are already familiar with the claim made in the *Timaeus* (48c) that 'Reason overrules Necessity by persuasion to achieve the best results'. Indeed, Necessity as it appears in the *Timaeus* indicates the Wandering Cause which originates in the physical properties of matter, and there is no indication in the *Timaeus* of the connection between 'necessity' and 'mathematics'. However, we should realise that the range of the concept 'necessity' is not restricted within the realm of Necessity as it appears in the *Timaeus*. We see, for example, in the *Republic* (V 458b), the concept 'necessity' is linked with 'geometry' and Plato contrasts the necessity of geometry with the necessity of love. Plato mentions: 'the necessities of love is far more influential and compelling than those of geometry' (*Republic* V 458b). In the *Laws*, the concept 'necessity' is linked with mathematical studies, and it is in the passage (*Laws* VII 818a-e) where the linkage is made. Let us examine this passage more precisely . First, at *Laws* 818a4-5 the Athenian Stranger says, 'for the majority of the people it will be proper to learn so much of the matter [the three mathematical subjects; τρία μαθήματα] as is indispensable (ὅσα αὐτῶν ἀναγκαῖα), and as it may truly be said to be shameful to the ordinary people not to know'. Here the term ' ἀναγκαῖα ' relates to '*indispensability* of the maqhvmata'. In the following statement given by the Athenian Stranger, we have a second, more complex, use of 'αναγκαιον': 'We simply cannot dispense with the *necessity* of the μαθήματα (τὸ ἀναγκαῖον αὐτῶν)' (818a7). As Taylor points out, the '*necessity* (τὸ ἀναγκαῖον)' at 818a7 can be construed as an echo of the '*indispensability* of the maqhvmata (ὅσα αὐτῶν ἀναγκαῖα)' as it appears at 818a4. Yet we should not fail to notice that the '*necessity* (τὸ ἀναγκαῖον)' at 818a7 may capture not merely the *indispensability* of μαθήματα but also the *necessity* inherent in mathematical knowledge. The mathematical necessity is then connected to the *Necessity* as it appears in the poem of Simonides: 'Not even God will ever be seen fighting against *Necessity* ' (818a7-818b3).[19] Although Simonides, the original author of the saying, does not use the word *Necessity* in the sense of *mathematical necessity*, the context in which the Athenian Stranger cites the dictum is a mathematical one, and that clearly opens up the possibility that we should think of the divine necessities here as including mathematical necessities. The Athenian Stranger goes on: 'No doubt he meant the *Necessity* which is *divine*, for if you understand the words of mere human necessities, like those to which men in general apply such sayings, they are far and away the silliest of sayings' (818b3-6). The nature of the necessities that are divine, not human, is clarified by the Athenian Stranger's answer to Cleinias' question at 818b. When asked what necessities belong to the μαθήματα that are not of that sort (i.e. merely human) but divine, the Stranger replies:

> Athenian Stranger: 'Those [divine necessities belonging to the maqhvmata], as I believe, which must be practised and learned by every god, daemon, and hero, if he is to be competent seriously to supervise mankind: a man certainly would be far from becoming godlike if he were incapable of learning the nature of one and of two, and of even and odd numbers in general, and if he knew nothing at all about counting, and could not count even day and night as distinct objects, and if he were ignorant of the circuit of the sun and moon and all the other stars'. (*Laws* VII 818c)

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19 This poem is also referred to in *Protagoras* 345d.

From this it appears (1) that the divine necessities are exemplified by knowledge of numbers and of astronomy, (2) he who knows the divine necessities can become godlike. The various branches of mathematics have to be distinguished, and they have to be learnt in due order -- again a matter of what necessity has laid down (818de).

As for (1), Plato does not mention the idea elsewhere in his corpus except in the possibly-inauthentic *Epinomis* where, as I shall discuss in the next section, special attention is given to knowledge of numbers and astronomy.[20] Here, we may point out a piece of evidence concerning the link between the *Epinomis* and the *Laws*.

As for (2) --'the person who knows the divine Necessities (i.e. the mathematical necessities) can become godlike'--, no similar claim can be found elsewhere in the Platonic Corpus except in the *Epinomis*.[21] Again, we may detect the link between the two works. (2) is suggestive of the famous dictum attributed to Plato that God is always doing geometry,[22] and the image of the Timaean Demiurge creating the universe by using 'numbers and shapes'.[23]

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20 Yet, why does not the divine necessity of the maqhvmata also entail knowledge of geometrical figures in terms of the analysis of shapes ? In considering this question, we should remember the suggestion made above that the art of measurement which deals with 'problems of essential nature of the incommensurable and the commensurable' can also be regarded as a part of arithmetic. If so, the vision of the universe that lies behind the framework of the three μαθήματα is not meant to exclude geometry, even though the emphasis is on numbers.

21 For example, *Epinomis* 992c6, we fine the expression 'ὁπόσοι θεῖοι', which indicates those having learnt μαθήματα.

22 Plutarch, *Quaest, conv*. 718b ff.

23 The motif 'to become godlike' also appears in *Laws* IV (716c):

> Athenian Stranger: 'In our eyes, God will be the measure of all things in the highest degree -- a degree much higher than is any "man" they talk of. So he who would be loved by such a being must himself become such to the utmost of his might.' (716d)

While the first statement in the above passage can be taken as a variation on the famous Protagorean dictum that man is the measure of all things, we should not fail to notice that the statement is immediately followed by the sentence which uses the motif 'to become godlike'. The phrase 'such a being' at 716c6 refers to a God which is the measure of all things. We can therefore say that 'becoming godlike' means 'becoming like a divine measure'. If we link this consequence with the idea in *Laws* VII that 'the person who knows mathematical necessity can become godlike', we might arrive at the idea that mathematical necessity correlates not merely with 'numbers' but

Evidently throughout this text the use of the term 'necessity' shifts. At one end of the spectrum it relates merely to the indispensable parts of education (818a4). At the other there is talk of a divine necessity and of studies that can make you godlike (818b3, 818b7-8, etc.). But what makes a human godlike is grasping mathematical truth -- especially those of arithmetic and astronomy --. Those, it would seem, are the necessities that not even a god would fight with.

But one interesting feature of the studies itemised at 818de is that while the study of numbers is clearly identified, and so too the study of day and night and the circuits of the sun, moon and stars, there is no explicit mention of geometry as such. This might be thought particularly surprising in view of the importance of shapes in the work of the Demiurge in the *Timaeus*. However, the arguments we have adduced earlier in this chapter on the various views to be found in Plato on the relationship between arithmetic and geometry suggest that it would be unwise to conclude, from this text in *Laws* VII, that geometry is meant to be excluded. We should recall that, in certain contexts, at least, the study of numbers embraces that of plane and solid geometry, or the latter studies can, in some sense, be reduced to the former. The clear reference to metrhtikhv at 817e shows that, at the outset of this discussion at least, the study of the measurements of length, surfaces and solid is clearly included. On the basis of that mention, we would take it that the studies that should be distinguished, and tacked in due order, at 818de implicitly include plane and solid geometry.

What emerges from *Laws* VII is a vision of the universe being governed by divine necessity, and that necessity relates to its mathematical structure. Higher education in mathematical studies may be confined only to a few, but its importance is still emphasised, for those studies enable one to become godlike and they give access to an understanding of the universe based on the two principles of number and measure.

Let us now explore how the two concepts 'number and measure' are used elsewhere in the *Laws*.　In *Laws* V, the concepts 'number' and 'measure' play important roles in the laws governing worldly and civic affairs. The positive use of 'number' and

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also with the concept 'measure'. The vision that the universe is governed by divine Necessity is thus grounded on the two principles of number and measure.

'measure' in political and economical areas seems to reflect the idea that 'wealth and property must be rated by the same scale' (728e) because 'fierce and dangerous strife occur concerning the distribution of land and money and the cancelling of debts' (736c). Therefore, assurance for the stability of the State is sought in an organised numerical system (747a7), which Taylor translates as 'numerical standardisation'.[24] Taking the example of the number '5040' which can be resolved into 59 factors, including all the digits from 1 to 12,[25] the Athenian Stranger states that 'this will give us our brotherhoods, wards, and parishes, as well as our divisions of battle and columns of route, and also the coinage system, dry and liquid measures, and weights -- to see, I say, how all these details must be legally determined so as to fit in and harmonise with each other' (*Laws* V 746d-e).[26] This statement clearly indicates that 'number' and 'measure' are key concepts in the political and economical domains. Moreover, we have at *Laws* V 741a the following statement:

> Athenian Stranger: 'My most excellent friends, be not slack to pay honour, as Nature ordains, to similarity and equality and identity and congruity in respect of number and of all that can produce fair and good effects. Above all, now, in the first place, guard throughout your lives the number stated [5040]; ...' (*Laws* V741a7-b3)

The above statement throws further light on the significance of number in civic affairs, and in particular of its relation to the concepts 'similarity and equality and identity and congruity'.[27] The Athenian Stranger seems to think that the role of number parallels 'all that can produce fair and good effects' which must of course include a God. The proverb 'even a God cannot fight against necessity', which we saw in *Laws VII* at 818a-b,

24 Taylor 1934: 130.
25 See also Burkert, 1972:431-432.
26 Cf. England,1921 : 540-541, Burkert, 1972.: 431-432.
27 We may also refer to *Gorgias* 508a, where the concept 'geometrical equality' appears.

appears just before the above statement (741a4-5). Referring to it, the Athenian Stranger explains that in order to maintain the appointed numbers of houses [5040], the people should be prepared to accept even an undesirable policy which a God might not want. The Athenian Stranger says: 'If our citizens are visited by a flood-tide, as we may call it, of disease, or by destruction in battle, and so reduced far below the appointed number by untimely deaths, we ought not, of our own free will, to introduce new citizens trained with a bastard education, --but with "necessity" (as the proverb says) "not even a God can cope".'(741a). Once again necessity is associated with number, and this certainly helps to underline the importance of number in the political domain. However, here the context is that it is when disastrous calamity affects the state, that drastic measures have to be taken to ensure its continued well being.

It is clear that in *Laws* V Plato pays special attention to the use of 'number' and 'measure' in civic affairs. He also refers to the importance of the study of numbers. The Athenian Stranger says, 'in domestic, and public life and in all the arts and crafts there is no other single branch of education which has the same potent efficiency as the study of numbers (πρός τε γὰρ οἰκονομίαν καὶ πρὸς πολιτείαν καὶ πρὸς τὰς τέχνας πάσας ἕν οὐδὲν οὕτω δύναμιν ἔχει παίδειον μάθημα μεγάλην, ὡς ἡ περὶ τοὺς ἀριθμοὺς διατριβή)' (747b). Accordingly, lawgivers are advised to study 'numbers'. The Athenian Stranger mentions that 'regarding numbers, every man who is making laws must understand at least this much, --what number and what kind of number will be the most useful for all States' (*Laws* V 738a); and also states that 'the lawgiver must keep all these [theories of numbers] in view and charge all the citizens to hold fast, so far as they can, to this organised numerical system' (747a). Some have used the label 'number mysticism' to describe the familiar relationship we find between number and political policies.[28] Yet that does not get us very far. Rather we should see all the passages in *Laws* V and VII, cosmological, educational and political, as making use of overarching concepts of 'number' and 'measure', to express the key ideas of limit and order.

28 Burkert 1972: 465-482.

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