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Article

An Evaluation of Temporal Credit-Saving Policies I

—Welfare Analysis in a Simple Financial Trading Model with Heterogeneous Risk-Neutral Traders—

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1 Introduction

This article is the first of the series in which the effect of temporal credit-saving policies by the government on the economic welfare of financial traders is analyzed.

In this first one, I show that, without any shortage of liquidity and any positive real effect, a temporal credit-saving policy that saves bankrupt equity traders do not improve the welfare of traders. The analysis is based on an extension of the 3-period general equilibrium model developed in Geanakoplos (2002) and Fostel and Geanakoplos (2008). In this model, a bond and an equity are traded by heterogeneous risk-neutral traders with an opportunity for leverage. I extend it by introducing temporal credit-saving policies and analyze their consequences on the economic welfare. The policy considered gives temporally credits to bankrupt equity traders. The government must finance the policy by levying taxes in the final period. The detail of the taxation, who pays how much, depends on the asset holdings in the new equilibrium, and the government does not announce it at the time of the policy implementation in order to maximize its temporal effect. The implementation of the policy shifts the equilibrium, and the payment of the equilibrium leverage contract into which equity holders have entered in the first period is adjusted accordingly. The content of the equilibrium leverage contract is to pay the worst value of the equity in the next period, and it is assumed to be maintained under a shift of equilibrium. By this adjustment, the increase of the equity price in the new equilibrium is paid off entirely to the promise holders, which works as a built-in compensation to the promise holders.

I show that, if implemented large enough in size, the policy can “illusionary” improve the welfare of traders at the state the policy is implemented. This improvement is “illusionary” because the tax to finance the policy cannot be foreseen by traders. I show that, in the final period, a group of bond holders are made worse off after paying the tax. More concretely, those who hold the bond regardless of the policy end up holding a risky asset whose payoff at the bad state becomes significantly lower, and those who are discouraged to hold the equity by its higher price end up avoiding bankruptcy at the bad state by sacrificing the payoff at the good state. Especially, if the state in the final period is bad, only those who are discouraged from holding the equity by the policy are possible beneficiaries. The policy also fail to achieve a Pareto optimal equilibrium allocation, since they do not alter the span of the financial market.

The results have two important implications for policy makers. First, the policy does not make sense if its intention is to improve the welfare of traders by revitalizing the equity trading when the bad-turn of the economy is highly expected in the real side of the market. When the economy turns out to be bad, not only that a group of bond holders is hurt, but also the potential beneficiaries of the policies are restricted to those who are discouraged from holding the equity, excluding the possibility that the equity holders are better off by a further redistribution of wealth without hurting the group of bond holders more, since the government cannot distinguish an individual bond holder from others. Secondly, there is no ex-ante ground to implement the policy since it does not achieve a Pareto optimal allocation, while there is a way to achieve it by implementing new market trading devices. In Fostel and Geanakoplos (2011), it is shown that introducing both tranching and naked CDS at the same time makes the financial market complete. Hence introducing them in the initial period makes the equilibrium allocation ex-ante Pareto optimal. These points further imply that, for implementing the policy to make sense, the government must intend to save "irresolute" from bankruptcy, whose decision to hold the equity is heavily influenced by the equity price level, by discouraging them from holding the equity, or have a conclusive evidence that the economy will almost surely turn out to be good.

We need to distinguish a temporal credit-saving for those caught in surprise by an unforeseen economic shock and that for those deliberately bankrupt by holding highly risky assets. In the articles, only the latter type is analyzed. The former is mainly to ease a very short-run liquidity shock to an economy in order to avoid chain reaction bankruptcies, and will have a positive welfare effect. The latter is implemented when a downward risk on the payoffs of risky assets is foreseen and some traders of risky assets, such as security companies, investment banks, and hedge funds, demand it vocally. A deliberate and sophisticated government will not take an action just for rewarding those demanding, but will implement the policy if it contributes to an over-all improvement of social and economic welfare, even if a part of such an improvement turns out to be an illusionary expectation. It is this possibility that the analysis in the articles examine.

The articles analyze two types of temporal credit-saving policies. The first type is to give a credit to bankrupt equity investors, which is analyzed in this article. The second type is to subsidize the payment of the promise issued by equity buyers if a bad state occurs in the next period, which is analyzed in the second article. Both actions must be financed in the final period. The government aims to bring a temporal welfare improvement at the state where the policy is implemented, without sacrificing anyone in the end after the cost is covered by tax. Intuitively, such an objective is hard to be fulfilled since, after the cost is financed, the total payoff available to traders remains the same as that without the policy, so the best it can achieve is to recover the allocation without the policy in the final period so that traders enjoy only an illusionary welfare improvement at the state where the policy is implemented. The results in the articles show that it is not possible to recover the original allocation after the tax is levied in the final period, though it is possible to have an illusionary improvement at the state where the policy is implemented. In other words, the government can achieve an illusionary welfare improvement only by sacrificing a real welfare in the future. Also, the results show that an illusionary welfare improvement at the time of the policy implementation can be achieved only if the sizes of the policies are big enough. This implies that an illusionary welfare improvement requires a large sacrifice of the real welfare in the future.

A quick glance at the quantity-easing policy in Japan will help making a point clear. This policy was implemented in March, 2001. At that time, both the financial market and the real economy had been stagnant with no expectation of recovery in the near future, and the need to finish the disposal of bad assets mounted after the corruption of the realty bubble had been advocated vocally. As clarified in Chapter 5 of Umeda (2011), the quantity-easing policy had been a topic in the Policy Board of BOJ from late 1998 in relation to an inflation target policy. The majority of the board had been reluctant to implement such a policy, stating unofficially that the market for short-term bonds maintained a sufficient level of liquidity, so that the quantity-easing would just result in replacing them with inside money, and have no real effect. However, the Japanese government was determined to take a move to finish off the disposal of bad assets, and the board had recognized that such a move might cause a new liquidity shock, which lead to the adoption of the policy. The restructuring of the banking sector was mostly finished in 2002, but the objective of the policy had been gradually shifted toward stimulating the real economy, and the board made a commitment to continue its quantity-easing policy in October, 2003. The policy had not been lifted until March 2006. The target size of the current account at BOJ was raised seven times during the period of policy installation, grew from the initial 0.5 trillion yen in 2001 to max. 3.5 trillion yen. To achieve this target, BOJ bought short-term bonds, commercial papers, even stocks and mortgage-backed securities from financial institutions. It also bought the long-term government bond in order to fill the gap between the target size of the account and supply of assets by those financial institutions. The empirical study on the effect of the quantity-easing policy is still ongoing, but preliminary results show that it is at least hard to observe a positive real effect, as stated in Chapter 10 of Umeda (2011). This implies that the quantity-easing affected only on the financial market. Arai (2012) finds that, during 2003–2006, the difference between the lending rate and deposit rate in the banking sector was steadily decreasing, and the small stocks, traded mainly in the 2nd section of TSE, had bubbled while the index for the big stocks, traded in the 1st section of TSE, had not moved drastically. The rise of the small stock price index shows a significant correlation with an index which represents a persistence of the expectation that the quantity-easing policy is sustained, so it is suspected that the lent money had been used for leverage to finance an investment to small stocks, as explained in Arai (2012). Also, the investment to foreign assets became very popular since the quantity-easing policy helped to maintain the near-zero rate on the deposit/bond, and the foreign asset markets had offered a much better look on the return. Though the empirical studies on this matter seem inconclusive at best, the data used for these studies is too restricted to capture all of capital movement, and there is a room to suspect that a big amount of lent money had been used for a foreign investment. Much of the investment in small stocks and the foreign investment has turned out to be bad later, and BOJ has returned to the zero-rate policy in December, 2008. On the part of the cost for the policy, the Japanese government has continued to spend heavily by issuing long-term treasury bonds with a high pace, and most of them had been bought by the banks in Japan. The treasury is backed by tax, so it means that a large population of Japanese nationals who choose to maintain their wealth solely by depositing to commercial banks owe a large share of the cost for the quantity-easing policy.

The analysis is based on a 3-period general equilibrium model, in that 2 assets, a bond and an equity, are traded. Traders are assumed to be risk-neutral, but have different prospects on the possibility of the bad-turn of the economy. Leverage opportunities are available for all traders, and the promise issued by equity buyers are traded naked. Assuming risk neutrality of traders makes all traders except a critical one hold exactly one type of asset, never mixed. This enables a simple categorization of traders by their asset holding, and the government can use this information to set up taxable bases. The model was first introduced by Geanakoplos (2002) as an example to a more general model with leverage opportunities and default. The model was elaborated further in Fostel and Geanakoplos (2008), and extended to incorporate tranches and CDS, more than 3 periods, and different uncertainty structures, in Fostel and Geanakoplos (2010) and Fostel and Geanakoplos (2011). The analysis in these papers take a focus on the equilibrium price dynamics and the resulting credit cycle. In Fostel and Geanakoplos (2012), the linear pricing formula and the "VAR=0" nature of the promise traded in equilibrium is proved to be true without risk neutrality of traders if the one-period uncertainty is binomial. However, the welfare effect of trading promises has been largely neglected in these researches. A positive effect on the economic welfare by introducing the possibility of default into a general equilibrium model was first found in Dubey et al. (1990) and Zame (1993). In these papers, default is allowed with a pre-specified penalty rate in terms of utility, which is not observable in the market. This characteristics makes the evaluation of policy, based on the observation of the market, impossible. Kaneko (2006) offers an analysis of economic welfare in a hypothetical trading system with the rate of successful contracting as a sole market variable. Since the temporal credit-saving policies are implemented in the standard market system, this model cannot be used to evaluate these policies.

This article is organized as follows. The section 2 describes the model used in the analysis. The section 3 reviews an equilibrium without any implementation of the policy, derived in Fostel and Geanakoplos (2008). The section 4 describes two types of temporal credit-saving policies to be analyzed in a series of articles, and the criteria to evaluate their welfare effects. The adjustment of a financial contract with a shift of equilibrium is discussed in the section 5. The analysis of the policy is given in the section 6. The section 7 evaluates the welfare effects of the policy. A conclusion and additional remarks on the result of this article is given in the section 8. Full analysis of a positive real effect by the policy is given in the appendix.

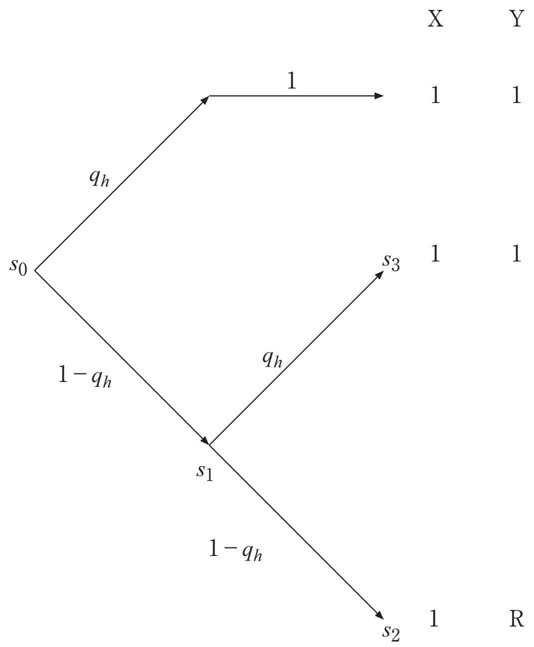
2 Model

The analysis is based on a simple 3 period model, introduced in Geanakoplos (2002) to analyze a pro-cyclic behavior of leverage. The following is the description of its model. The readers who are interested in the analysis of leverage cycle are referred to Geanakoplos (2002), Fostel and Geanakoplos (2008), Fostel and Geanakoplos (2010), and Fostel and Geanakoplos (2011). The readers whose focus is on the pricing of promises are referred to Fostel and Geanakoplos (2012).

The current period is zero, with a unique state s_0. In the period 1, a “good” state or a “bad” state may occur. In the “good” state, all uncertainty in the economy is resolved. In the “bad” state, two states, again called as a “good” state and a “bad” state, may follow in the period 2. I call the “bad” state in the period 1 s_1, the “bad” state in the period 2 s_2, and the good state in the period 2 s_3. The event tree of the economy is shown in Figure 1.

There is a commodity for consumption traded in spot markets, hidden behind the financial economic model by tradition. All market val-

Figure 1: Uncertainty of the economy



ues are measured against the value of the commodity, so that I can safely assume that the value of the commodity is always 1. Note that there is no real effect derived from nominal values of goods and assets, since there is no restriction on the market liquidity of goods and assets, which is traditionally the case of general equilibrium models.

There are two assets traded in the market, a bond X and an equity Y. The bond X is risk-less, while the equity Y has a risk of devaluation if a "bad" state occurs. In the period 2, the payoff of the bond X is 1 in any state, while the equity Y pays 1 at all states other than s_2, where it pays $R<1$.

There is a continuum of traders, distributed uniformly on [0, 1]. All traders are assumed to be risk neutral, but different traders have different subjective probabilities (beliefs) that a "good" state will occur. The traders are aligned increasingly by the optimism, so there is an increasing continuous function q: $[0, 1] \to [0, 1]$ such that the subjective probability of the trader h that a "good" state will occur is the value of q at h, which I write as q_h.

The assumption of risk neutral traders with heterogeneous beliefs is obviously quite extreme, but works well for analyzing equilibria with bankruptcy since the traders can bet on their most preferable state so that they will go bankrupt quite easily. In the analysis, the assumption makes it easier to identify a group of traders that is the target of a temporal credit-saving policy.[1]

At the current state s_0, each trader has one bond and one equity. Hence the total supply of the bond and the equity is 1 at any states in the period 0 and the period 1.

The model assumes that traders can take advantage of leverage on the equity Y. A leverage is to borrow by promising to pay back later when buying risky assets. The promise issued for leverage in the binary uncertainty setting of this model can be expressed as (j, j), where j is the amount to pay back in the next period promised by borrowers. In U.S., the promise is a non-recourse contract so that, if the gain from holding risky assets with leverage is less than the amount promised, the borrowers pay only that gain and default the rest. In Japan, the promise is a recourse contract so that the borrowers must pay the amount they promised regardless of the gain they receive by holding assets with leverage. Each borrower is allowed to choose the level of promise when he decide to take advantage of leverage opportunity. Hence, potentially, the model does not exclude the possibility of multiple levels of promises chosen by traders, and of default when contracts are non-recourse. However, as Geanakoplos (2002) and Fostel and Geanakoplos (2008) shows, in this simple model, all traders choose the same level of promise in equilibria, and there is no default. In Fostel and Geanakoplos (2012), it is proved that this remains to be true without assuming risk-neutrality of traders, if the underlying uncertainty of the economy is structured binomially. Hence, whether the leverage is recourse or non-recourse does not matter.

The model also assumes that traders cannot take a short position

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1　In the real world, such traders are likely to be small future traders and/or foreign exchange traders, whose influence on the financial market would be limited at best, compared with that by risk-averse institutional traders. Hence the correct modeling would be assuming the homogeneous belief (subjective probabilities are the same) and aligning traders with respect to the degree of risk aversion, or allowing any combination of a subjective probability and a degree of risk aversion.

on either of the bond or the equity. This assumption is not adopted by the standard financial economics model. Fostel and Geanakoplos (2011) gives an explanation to defend this assumption, but I would like to give a different line of defense here. It has been shown in the numerous researches in 80's and 90's that the existence of an economic equilibrium in a financial economics model requires either that the trading strategies satisfy a "nice" moment condition (typically a kind of L^2 structure), or that short sales have a finite bound. The moment condition to make price-quantity relation as an inner product may be handy in mathematics, but is purely structural, meaning that there is no solid justification in terms of traders' economic behavior. The exclusion of the so-called "Dutch doubling-strategy" without a bound on short position is achieved by the finiteness of trading opportunities in discrete-time trading models. However, this finiteness is just a loose proxy to the real world trading, and the result that solely depends on this assumption should be taken with a caution. On the other hand, among others, Hart (1975) shows that the possibility of infinite short sales destroys the existence of equilibrium. In the real world trading, the so-called short position in financial assets is attained only by borrowing from the existing long position, and taking a huge short position requires a public report, which can be scrutinized later. In addition, most of trading practices set a hair-cut line at which an obligation/debt, such as a short position, is forced to clear out. So, assuming a finite bound on short position seems natural in economic analysis. Surely, this bound need not be zero as assumed in this model. But, if traders are allowed to be short on bonds with a significant amount, then there is no need of leverage on the equity, the financial market is effectively complete, and the welfare analysis becomes quite obvious. Namely, since the financial market is complete, the equilibrium allocation is Pareto optimal, and any credit-saving policies other than the redistribution of income have a risk of making the allocation sub-optimal.[2]

I assume that traders with the same trading strategy are symmetric. All bond buyers at s_0, having the same income, purchase the same amount of the bond-equivalents (*X* and a non-defaulting promise). The same for all equity buyers with leverage at s_0. At s_1, only bond buyers at s_0 are active, and they have the same income, hence the symmetry holds again. Assuming the symmetry simplifies the analysis since all bond/promise buyers have the same income at s_1 Namely, the effect of an equity price increase is uniform among bond/promise holders, so that there are no relative winners and losers inside this group.

3 Equilibrium Without Temporal Credit-Saving Policy

Geanakoplos (2002) solves for a unique equilibrium without any policy implementation in this model. I reproduce its result in order to give a starting point of my analysis. For a complete proof and an extension to n-period model with $n>3$, see Fostel and Geanakoplos (2008) and Fostel and Geanakoplos (2010).

In a unique equilibrium, there are bankrupt traders in the period 1 and 2. At the state s_0, all traders buying the equity *Y* take advantage of leverage by issuing the promise equal to the price of the equity at

2 The redistribution of income is just the policy to choose one among Pareto optimal allocations, but I do not intend to explain who should be saved by sacrificing others in my analysis, so I do not pursue this line.

the state s_1. There is a trader h_0^*, called as a marginal buyer of the equity in Geanakoplos (2002), such that those in $(0, h_0^*]$ buy the bond X and the promise, while those in $(h_0^*, 1]$ buy the equity with leverage. The trader h_0^* is indifferent between buying the bond/promise and buying the equity with leverage. In this article, it is assumed that he buys the bond/promise. At s_1, those who bought the equity at s_0 go bankrupt. They sell all the equity in the market, pay the promise, and cease to exist as traders. Those who bought the bond/promise continue to trade. All traders buying the equity take advantage of leverage by issuing the promise equal to R. There is a trader $\bar{h}_1$ $(\bar{h}_1 < h_0^*)$ such that traders in $[0, \bar{h}_1)$ buy the bond/promise while those in $(\bar{h}_1, h_0^*]$ buy the equity with leverage. The trader $\bar{h}_1$ is indifferent between buying the bond/promise and buying the equity with leverage, and assumed to buy the bond/promise. In the period 2, those who bought the equity at s_1 are bankrupt at s_2.

The equilibrium prices and allocations are determined as follows. Let me denote the price of the equity at s_k by $p(k)$, and the price of the promise j at s_k by $\pi_j(k)$, where k is either 0 or 1.

At s_0:

Take $p(1)$ as given. The only promise issued is $j = p(1)$. Since its payoff is co-linear with that of X, and both of them are traded in positive amounts, the arbitrage makes $\pi_j(0) = p(1)$ in equilibria. The trader h_0^* is indifferent between buying the bond/promise and buying the equity with leverage, so, the marginal utility of expenditure on the bond/promise and that on the equity with leverage must be equal. Note that the trader h_0^* will be buying the equity with leverage at s_1, and there the only promise issued is j

$=R$. This implies that the following equation must be satisfiued,

$$\frac{q_{h_0^*}(1-p(1))}{p(0)-p(1)}=q_{h_0^*}+(1-q_{h_0^*})\frac{q_{h_0^*}(1-R)}{p(1)-R}. \tag{1}$$

The market clearing condition on Y makes the following equation being satisfied,

$$(1-h_0^*)\frac{1+p(0)}{p(0)-p(1)}=1. \tag{2}$$

(By the Walras law, the bond market is automatically cleared once the equity market is cleared.) Solving these two equation for $(h_0^*, p(0))$ determines them as a function of $p(1)$.

At s_1:

Take $(h_0^*, p(0))$, a function of $p(1)$, as given. All traders $h\in(h_0^*, 1]$ goes bankrupt, selling all Y they have and passing the revenue to buyers of the promise. All traders $h\in[0, h_0^*]$ secure the same positive income $\frac{1+p(1)}{h_0^*}$, and continue to trade. The only promise issued is $j=R$. Since its payoff is identical to that of X, and both of them are traded in positive amounts, the arbitrage makes $\pi_R(1)=R$ in equilibria. The trader $\bar{h}_1$ is indifferent between buying the bond/promise and buying the equity with leverage, so that the following equation is satisfied,

$$\frac{q_{\bar{h}_1}(1-R)}{p(1)-R}=1. \tag{3}$$

The market clearing condition on the equity makes the following equation being satisfied,

$$(h_0^*-\bar{h}_1)\frac{1+p(1)}{h_0^*}\frac{1}{p(1)-R}=1. \tag{4}$$

Since q is a function of h and $(h_0^*, p(0))$ is a function of $p(1)$, solving these two equations for $(\bar{h}_1, p(1))$ determines the equilibrium values of these variables, and those of $(h_0^*, p(0))$.

The reason why only one promise is traded and it allows no default, is based on the comparison of marginal utilities of expenditure on X, the promise, Y with no leverage, and Y with leverage, for all traders. A general proof without the assumption of risk-neutrality is given in Fostel and Geanakoplos (2012).

In this economy, the marginal utility vectors of traders are $\{(q_h, 1-q_h)\}_{h\in[0,1]}$. They span R^2_+ positively. This means that the equilibrium allocation is not Pareto optimal unless the financial market is complete without short sale. The completeness of the financial market is achieved only when all Arrow securities are available to all traders. However, at s_1, the bond/promise buyers cannot have an access to the Arrow security on s_2 since they cannot short the equity.[3]

The following argument shows that the equilibrium allocation without temporal credit-saving policies is indeed Pareto suboptimal at s_1. The idea is to allocate two Arrow securities properly to all traders. The total supply of the Arrow security on s_2 is $1+R$, while that of the Arrow security on s_3 is 2. Note that, in equilibrium, each equity buyer holds the Arrow security on s_3 by the same amount, $\frac{1}{q_{\bar{h}_1}}\frac{1+p(1)}{h_0^*}$. Construct a new allocation as follows. First, allocate exactly the same amount of the Arrow security on s_3 as that in the equilibrium alloca-

3 On the other hand, the equity buyers have an access to both the Arrow security on s_2 and that on s_3, thanks to the opportunity for leverage.

tion to each trader in $(\bar{h}_1, h_0^*]$. By the market clearing condition on the equity, this uses up $(1-R)$ of Arrow securities on s_3, out of 2. Hence both Arrow securities are available to traders in $[0, \bar{h}_1]$ by the same amount, $(1+R)$. Consider the economy in which two Arrow securities are traded by the group $[0, \bar{h}_1]$, where each member is endowed with $\frac{1+R}{\bar{h}_1}$ of both Arrow securities, and obtain a competitive equilibrium of this economy in which prices of two securities are normalized to sum up to 1. By the market clearing condition on the bond/promise in the original equilibrium, each trader has the income $\frac{1+p(1)}{h_0^*}$. In the equilibrium, there exists a trader h' who is indifferent between Arrow securities, all traders in $[0, h')$ buy the Arrow security on s_2, and all traders in $(h', \bar{h}_1]$ buy the Arrow security on s_3. (The equilibrium price of the Arrow security on s_2 is $(1-q_{h'})$, while that of the the Arrow security on s_3 is $q_{h'}$.) The competitive equilibrium makes every one, except h', better off than holding the endowment. Since the endowment is exactly the bond/promise, it means that this competitive equilibrium allocation improves the original equilibrium allocation for the trader group $[0, \bar{h}_1]$. Hence, by allocating the new equilibrium allocation to $[0, \bar{h}_1]$, we obtain an allocation in the original economy that improves the equilibrium allocation at s_1.

4　Types of Temporal Credit-Saving Policies

In a series of articles, I consider two types of temporal credit-saving policies implemented at s_1.

Type 1 Give a positive income symmetrically to each bankrupt trader.

Type 2 Promise to pay the difference between the issued promise on Y and R at s_2.

It is assumed that an action by the government does not create a new credit, so that both policies must be totally backed by tax in the period 2. Both policies are of a temporary nature, and the resource to finance those increased credits is not made clear, rather deliberately, at the time the policies are implemented. The government suddenly collects the cost by levying taxes after all securities pay out and before the consumption occurs in the period 2, based on the observable characteristics of traders in the financial market. The government does not announce its tax rule until it is actually enforced. Traders are fooled since they cannot know how the government will tax them based on their information available in the market at the time they decide their trading strategies. Traders may expect that they will be taxed later, but the tax rule will depend on the pools of traders holding the same security at s_1 in an equilibrium, which no trader has an ability to anticipate before the equilibrium trades actually occur.

How the policies make sense to the financial economy? It is assumed that the traders are outcome-oriented in the sense that they regard the government intervention as acceptable if their conceived utilities are at least not worse off. Namely, if the government can illusionary make them better off at s_1 and keep them not worse off after tax in the period 2, these policies will be acceptable, or even welcomed.

The Pareto improvement is not enough for a policy to be sound, however. If the equilibrium allocation after the government intervention is still Pareto suboptimal, some traders will not be satisfied, while the others are indifferent. Then a further intervention by the government may be demanded by a group of investors collectively. I require that a satisfactory policy must achieve a Pareto optimal allocation. This actually means that the policy must recover the completeness of the financial market.

5 Adjustment of the Equilibrium Financial Contract With a Shift of Equilibrium

The implementation of polices at s_1 triggers a shift of the equilibrium afterward. The promise issued by the equity holders at s_0 has not yet paid off at the time of policy implementation, so that the payment may change according to this shift. This is so since the decision by equity holders to issue the promise for leverage at s_0 has been based on the common expectation of the equity value at s_1, which may change by the implementation of the policy.

It is not a common economic practice that the nominal value of the payment is maintained blindly under such a circumstance. The reason why the equity holders at s_0 issued the promise $p(1)$, among all other available leverage contracts, was precisely that it was a common knowledge among traders that $p(1)$ would be the equity price at s_1 in "the" equilibrium. Hence this nominal payment value loses its ground entirely once it becomes a common knowledge among traders that it is not the equity price realized at s_1, so that traders have no incentive to carry out the promise "as is".

Traders face a difficulty to evaluate the payment value since it is simultaneously determined with the new equilibrium. If there is enough time to rearrange the contract through a staged process, both the new equilibrium and the new payment are determined subject to the design of this process. The outcome will create winners and losers among traders, so that traders are drawn into a competition on the procedure to design a process of contract rearrangement. The equilibrium outcome of this competition will create winners and losers among traders again, so that they are now drawn into a competition on a procedure to design a procedure to design a staged process for a rearrangement of the financial contract. And so on. Such a competition on procedures may run indefinitely if the time to realize trades in the market is neglected, or no solution to stop the competition immediately is proposed. The market cannot wait the outcome of rearrangement indefinitely, so a solution to end such an infinite sequence of competitions is called for.

It is somewhat common but not good as an economic practice that an authority working as a neutral arbitrator other than the market system, such as a legal arbitration by the court, writes the design of a staged process for a rearrangement. Such a solution is typically rationalized by the bounded rationality of traders, in the sense that no traders can carry out the computation of rational strategies in a series of competitions that may run indefinitely. However, that is a grossly misplaced use of the notion of the bounded rationality. The market system involves all economic agents, including the arbitrator, so that handing an authority to design the process to this particular economic agent gives him an outstanding privilege to pursue his economic interest. If he can manage to present various designs, he will be able to choose one that is most economically beneficial to him. This means that the bounded rationality of financial traders is used as a threat to achieve an enhancement of the position of a particular economic agent to pursue his economic interest rationally. For example, since the federal court system is a part of the government in a broad sense, the economic interest of the arbitrator may be identified with that of policy makers who create "the" controversy itself. Then, a coalition of the arbitrator and the policy makers can exploit maximally from financial traders by proposing an all-or-nothing choice on a package of a financial policy and an arbitration process, citing traders' bounded rationality as a threat. To prevent this to happen, a trade-off in economic interest between policy makers and the court must be introduced, which eats up the resources in the broad economy not modeled in the simple financial model we are working on, and drives down the market economic welfare. The bounded rationality must be imposed on all economic agents involved in that market incident, including the arbitrator and the government, equally, if it is imposed at all.

A realistic solution, which is also sound as an economic practice, is to maintain the content of the financial contract, not the nominal value. The equilibium promise issued by the equity holders at s_0 is to pay the least value of the equity price realized in the next period. When this value changes by a shift of the equilibrium, the payment is automatically adjusted accordingly. This solution is surely bounded rational since it cannot be fully rationalized. For example, if the equity price at s_1 rises in the new equilibrium, all equity holders at s_0 gain by sticking to the nominal value of the payment. However, results in the next section guarantee that the solution can be implemented with the type 1 policy without any cost if the government can make the payment of the new equilibrium equity price as a precondition to give credits to bankrupt traders. Without the implementation of the policy, equity holders are sure to go bankrupt so that they have no choice but to obey. Since the additional credit is given to them simultaneously with the execution of trades in the market, it is impossible for the traders to receive the credit first by pretending to obey and default the difference to the nominal value if it is positive. We actually will see that the equity price rises in the new equilibrium. Hence, not only equity holders but also bond holders gain in their temporary income at s_1. We will also see that the government is also satisfied since more traders end up holding bond/promise so that it can increase a taxable base in the period 2. Hence no economic agent loses temporary at s_1. On the other hand, with the type 2 policy, in order to implement the solution, it is necessary to clear the market account of bankrupt traders using the nominal value of the promise first, and pool its total amount for the final payment to the promise holders, before the policy is implemented. Otherwise the equity traders will refuse to pay more than the nominal value on their promises, since they know that they are not saved by the policy. Hence, presumably, implementing the solution with the type 2 policy would incur a social cost.

In the following analysis, it is assumed that the solution to maintain the content of financial contracts under a shift of equilibrium is implemented in the financial economy without any cost. The assumption maximizes the room for a temporary improvement of the economic welfare under the policy, so that a negative result with this assumption means that the policy is not worth implementing.

6 Welfare Effect of the Type 1 Temporal Credit-Saving Policy

In this section, the model is solved for an equilibrium when the temporal credit-saving policy of type 1 is implemented, and evaluated in terms of economic welfare.

Let M be the total amount of credit the government gives equally to bankrupt traders at s_1. Since the model does not have money, this is achieved by an increase of the bond supply by M, and and distributing them to the bankrupt traders as endowments.

Each equity holder h in $(h^*, 1]$ now has a positive income $\frac{M}{1-h_0^*}$ at s_1. The income of each bond/promise holder h in $[0, h_0^*]$ is also increased. These traders have equal amount of promises issued at s_0. The assumption made earlier says, though the promise is denoted as $p(1)$ mathematically without the policy, it actually promises the worst value of the equity in the period 1. Since some equity holders, now with positive incomes, will continue to hold the equity, the new equilibrium equity price $p^*(1)$ will be higher than $p(1)$. So, the promise now pays $p^*(1)$, which is higher than the original payment $p(1)$. Therefore, each bond trader h in $[0, h^*]$ has the income $\frac{1+p^*(1)}{h_0^*}$, increased from the original $\frac{1+p(1)}{h_0^*}$.

Let's denote the promise at s_1 used for leverage by equity traders as j, where $j \in [0, 1]$. The promise j pays the amount min (j, R) at s_2 and j at s_3. Let $\pi(j)$ be the price of the promise j.

Let h_1^* be such that $p^*(1) = q_{h_1^*}1 + (1-q_{h_1^*})R$. By solving for $q_{h_1^*}$,

$q_{h_1^*} = \dfrac{p^*(1) - R}{1 - R}$. The trader h_1^* is the marginal buyer of the equity with the policy.

For each trader h, the marginal utility of expenditure on the equity with leverage is $\dfrac{q_h(1-j) + (1-q_h)\,max(R-j,\ 0)}{p^*(1) - \pi(j)}$, that on the equity without leverage is $\dfrac{q_h + (1-q_h)R}{p^*(1)}$, that on the promise j is $\dfrac{q_h j + (1-q_h)\,min(j,\ R)}{\pi(j)}$, and that on the bond is 1. A comparison on these marginal utilities of expenditure and the no-arbitrage in equilibria show a key observation, that only the promise R is traded in equilibrium, all $h \in [0,\ h_1^*]$ buy the bond and the promise, and all $h \in (h_1^*,\ 1]$ buy the equity with leverage. The result is the same as that in Geanakoplos (2002), since the wealth level of a trader do not affect his marginal utility of expenditure, by the assumption that traders are risk neutral. It is also a trivial application of the general result in Fostel and Geanakoplos (2012).

To determine an equilibrium, we need to calculate h_1^* and $p^*(1)$. They are determine by two equations, one is the definition of h_1^*, the other is the market clearing condition on either the equity Y or the bond and the promise R. By the Walras law, one of the market clearing condition is redundant, so that we have two equations to determine two unknowns.

For the market clearing condition, since traders in $[0,\ h_0^*]$ and traders in $(h_0^*,\ 1]$ have different incomes, there are two cases to be considered. One is the case $h_1^* \leq h_0^*$, the other is the case $h_1^* > h_0^*$. In the former case, all bond/promise buyers have the same income, while the equity buyers include all equity buyers at s_0 who are saved by the policy. In this case, the market clearing condition on the bond/promise is

$$h_1^*\left(\frac{1+p^*(1)}{h_0^*}\right)=1+M+R,$$

while the market clearing condition on the equity is

$$M+(h_0^*-h_1^*)\left(\frac{1+p^*(1)}{h_0^*}\right)=p^*(1)-R.$$

In the latter case, all equity traders are those saved by the policy, while the bond traders are mixed. In this case, the market clearing condition on the bond/promise is

$$1+p^*(1)+(h_1^*-h_0^*)\left(\frac{M}{1-h_0^*}\right)=1+M+R,$$

while the market clearing condition on the equity is

$$(1-h_1^*)\left(\frac{M}{1-h_0^*}\right)=p^*(1)-R.$$

By solving for $p^*(1)$ and $q_{h_1^*}$, we have the following system of equations.

When $h_1^*\leq h_0^*$:

$$\begin{cases} q_{h_1^*}=\dfrac{1}{1-R}\,[\dfrac{h_0^*(1+M+R)}{h_1^*}-(1+R)], \\ p^*(1)=\dfrac{h_0^*(1+M+R)}{h_1^*}-1. \end{cases}$$

When $h_1^*>h_0^*$:

$$\begin{cases} q_{h_1^*}=\dfrac{1}{1-R}\left[\dfrac{1-h_1^*}{1-h_0^*}M\right], \\ p^*(1)=\dfrac{1-h_1^*}{1-h_0^*}M+R. \end{cases}$$

Note that, in both cases, the r.h.s. of the equation for $q_{h_1^*}$ is increasing with respect to M and decreasing with respect to h_1^*. Since q is increasing, this implies that the equation has at most one solution, and that the solution h_1^* is increasing with respect to M. Obviously, if $M=0$, then the solution coincides with that without policy, $\bar{h}_1$. Since $p^*(1)=q_{h_1^*}(1-R)+R$, $p^*(1)$ is increasing with respect to M. Note that $h_1^*=h_0^*$ if and only if $q_{h_0^*}=\dfrac{M}{1-R}$ and that $p^*(1)=M+R$ in this case. Therefore,

$$M \lesseqqgtr (1-R)\,q_{h_0^*} \Leftrightarrow h_1^* \lesseqqgtr h_0^*.$$

Now I analyze the welfare effect of the policy. I calculate in two different cases, separately.

<u>**Case 1**: $0<M<(1-R)q_{h_0^*}$</u>

First, I calculate expected utilities achieved by traders conditional on the occurrence of s_1, and compare them with those without policy. Note that $\bar{h}_1<h_1^*<h_0^*$ in this case. The result is summarized in Table 1.

For $h\leq\bar{h}_1$, the conditional expected utility is increased simply because they are bond holders regardless of the policy, and their income is increased. For $h\in(h_0^*, 1]$, it is also increased simply because any opportunity of trading is better than bankruptcy.

For $h\in(h_1^*, h_0^*]$, the ratio of the conditional expected utility with policy to that without policy is,

$$\frac{\left(\dfrac{1+R}{1-R}\right)\dfrac{1}{q_{h_1^*}}+1}{\left(\dfrac{1+R}{1-R}\right)\dfrac{1}{q_{\bar{h}_1}}+1}<1.$$

Table 1: A comparison of expected utilities when $M<(1-R)q_{h_0^*}$

Trader Group	Security Holding w.o. policy→w. policy	E.U. at s_1 w. Policy	E.U. at s_1 w.o. Policy
$[0, \bar{h}_1]$	Bond→Bond	$\frac{1+p^*(1)}{h_0^*}$	$\frac{1+p(1)}{h_0^*}$
$(\bar{h}_1, h_1^*]$	Equity→Bond	$\frac{1+p^*(1)}{h_0^*}$	$\frac{q_h}{q_{\bar{h}_1}}\frac{1+p(1)}{h_0^*}$
$(h_1^*, h_0^*]$	Equity→Equity	$\frac{q_h}{q_{h_1^*}}\frac{1+p^*(1)}{h_0^*}$	$\frac{q_h}{q_{\bar{h}_1}}\frac{1+p(1)}{h_0^*}$
$(h_0^*, 1]$	Equity→Equity	$\frac{q_h}{q_{h_1^*}}\frac{M}{1-h_0^*}$	0 (bankrupt)

Therefore their conditional expected utilities are decreased by the policy.

For $h\in(\bar{h}_1, h_1^*]$, the ratio of the conditional expected utility without the policy to that with the policy is,

$$\frac{q_h}{q_{\bar{h}_1}}\left(\frac{\frac{1+R}{1-R}+q_{\bar{h}_1}}{\frac{1+R}{1-R}+q_{h_1^*}}\right).$$

The ratio is clearly continuous and increasing with respect to h. Since $q_{\bar{h}_1}<q_{h_1^*}$, the ratio is less than 1 when $h=\bar{h}_1$ and more than 1 when $h=h_1^*$. Hence, by the intermediate value theorem, there exists a $h'_1\in(\bar{h}_1, h_1^*)$ such that the conditional expected utility for the trader h'_1 remains the same after the introduction of the policy. For $h\in(\bar{h}_1, h'_1)$, the conditional expected utility is increased by the policy, while it is decreased by the policy for $h\in(h'_1, h_1^*)$.

The interval (h'_1, h_0^*) constitutes an open neighborhood of h_1^*. So, we can say that there is a neighborhood of h_1^* such that temporary welfare of traders in it is made worse by the temporal credit-saving policy of the type 1. The economic intuition for this result is quite clear. These traders were bond/promise buyers at s_0, and would be sufficiently willing to buy the equity with leverage at s_1 if the policy was not implemented. The incentive to do so would be given by the lowered price of the equity by the bad news, which would improve the marginal utility of expenditure on the equity with leverage. But, the policy makes the equity price higher, so the marginal utility of expenditure on the equity is lowered. By the definition of h'_1, all traders in $(h'_1, h_0^*]$ are improved by buying the bond/promise with the policy, and it is the best choice for them. For $h \in (h_1^*, h_0^*]$, buying the equity with leverage is still better, but the marginal utility of expenditure on the equity is lowered compared with that without the policy. Note that the argument includes a positive effect on the conditional expected utility by the increase of their income. The fact that $h'_1 < h_0^*$ means that $M < (1-R)q_{h_0^*}$ is not enough for this positive income effect to offset the negative effect on the return of the equity.

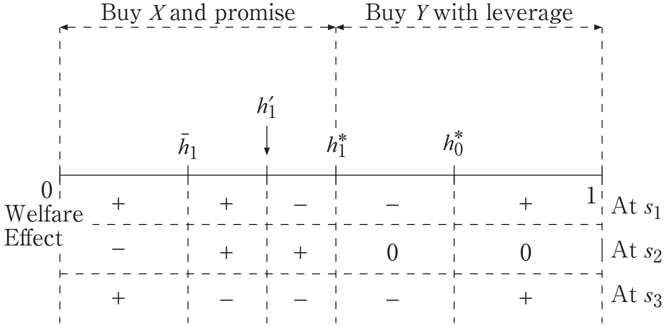
I have shown that the type 1 policy with $M < (1-R)q_{h_0^*}$ cannot Pareto improve the temporary welfare of traders at s_1. As I explained in the previous section, this temporary welfare is illusionary, since traders do not take their tax burden in the future period into account. Now the reality must catch the traders in period 2. The government's objective is to finance M from the beneficiaries of the policy without making the payoff of any trader worse than that before the implementation of the policy, at both

s_2 and s_3. At s_2, all traders saved from the bankruptcy is bankrupt again, so that only the bond buyers at s_0 can be taxed. If the government can identify h'_1, the maximum amount that the government can collect is $\bar{h}_1 \times \left(\frac{1+p^*(1)}{h_0^*} - \frac{1+p(1)}{h_0^*}\right) + (h'_1 - \bar{h}_1)\frac{1+p^*(1)}{h_0^*}$. By a simple calculation using the market clearing condition of the bond/promise with or without the policy, this amount is equal to $\frac{h'_1}{h_1^*}M - \left(1 - \frac{h'_1}{h_1^*}\right)(1+R)$, which is surely less than M. More serious problem for the government is that, since it does not know the function q, there is no way that the government recognizes h'_1. The government therefore has no other way than taxing all bond buyers at s_1. Also, the government cannot know $\bar{h}_1$, since the original equilibrium trading does not happen because of the implementation of the policy.[4] Hence the government has no other choice but taxing $\frac{M}{h_1^*}$ to each bond/promise buyer. By the market clearing condition on the bond/promise, the payoff (utility) for a bond/promise buyer after tax is $\frac{1+R}{h_1^*}$. For a trader in $[0, \bar{h}_1]$, it is less than the payoff without the policy, which is $\frac{1+R}{\bar{h}_1}$, also by the market clearing condition on the bond/promise. Traders in $(\bar{h}_1, h_1^*]$ are better off since they avoid the bankruptcy.

4　After the implementation of the policy, the original equilibrium at s_1 becomes counter-factual, something that did not happen. Since the government does not know q, it has no ability to calculate $\bar{h}_1$.

At s_3, the situation looks better since the government can levy all payoffs of traders saved from the bankruptcy at s_1. It amounts to $M\times\frac{1-R}{p^*(1)-R}$, which exceeds M. Hence the government can finance the policy from those saved by the policy and still leave them positive payoffs. However, there are traders whose utilities (payoffs) are made worse by the implementation of the policy. These are traders in $(\bar{h}_1, h_0^*]$. For those in $(h_1^*. h_0^*]$, this must be the case since their conditional expected utility at s_1 is lowered and they are bankrupt at s_2 regardless of the policy. For those in $(\bar{h}_1, h_1^*]$, the utility (payoff) at s_3, $\frac{1+p^*(1)}{h_0^*}$, is less than that if they bought the equity with leverage, which is less than that without the policy. Hence their utility (payoff) is also decreased by the policy. Beyond financing M, the government may try to compensate for them at s_3, but such an action requires a knowledge of $\bar{h}_1$, which the government has no way of knowing. In particular, taking away all of the payoffs from traders in $(h_0^*, 1]$ and redistributing to them after subtracting M never works. There are two reason for this. First, the total payoff available at s_3 after tax is the same as that without the policy (equal to 2), while payoffs to traders in other categories are either unchanged (for traders in $(h_0^*, 1]$, remaining bankrupt) or increased (for traders in $[0, \bar{h}_1]$). Secondly, the government cannot identify the trader group $(\bar{h}_1, h_1^*]$, so that there is no way for the government to redistribute to that group. So, at s_3, the government can finance M with those who would have been bankrupt and those who would

Figure 2: Welfare effect of the small scale policy



have been buying the bond/promise at s_1 still being improved, but the victims of the policy, traders in $(\bar{h}_1, h_0^*]$, cannot be fully saved, and those in $(\bar{h}_1, h_1^*]$ are sure to be left being worse off. Over all, the bond/promise buyers are victimized by the implementation of the policy. The analysis is summarized in Figure 2.

<u>Case 2: $M \geq (1-R)q_{h_0^*}$</u>

As is shown before, $h_1^* \geq h_0^*$.

Let's consider the conditional expected utilities of traders at s_1, and compare them with those without policy. Let the trader $h'_1 \in (\bar{h}_1, h_1^*)$ be defined as before by

$$\frac{q_{h'_1}}{q_{\bar{h}_1}}\frac{1+p(1)}{h_0^*}=\frac{1+p^*(1)}{h_0^*},$$

or, equivalently, by $q_{h'_1}=q_{\bar{h}_1}\times\dfrac{\dfrac{1+R}{1-R}+q_{h_1^*}}{\dfrac{1+R}{1-R}+q_{\bar{h}_1}}$. Note that h'_1 increases as M increases. If $h'_1 < h_0^*$, then conditional expected utilities of the traders in $(h'_1, h_0^*]$ are lowered by the policy. Hence the government has an incentive to set M so large that $h'_1 \geq h_0^*$. A calculation shows that, if the government sets M so that $h'_1 = h_0^*$, then the conditional expected utilities of all traders except h_0^* are improved. Table 2 summarizes the result of this calculation.

Considering the necessity to finance later, the government does not have any incentive to set M higher than the level consistent with $h'_1 = h_0^*$. I denote this level of M as M^*. An economic intuition for the improvement is trivial. All traders in $(h_0^*, 1]$ would have been bankrupt if the policy M^* was not implemented, so they are improved just by getting positive incomes. All traders in $(\bar{h}_1, h_0^*]$ are improved since the relatively large increase of their income by the policy makes the expected payoff on the equity without the policy unattractive.

I have shown that, with the policy M^*, the government can achieve a strict Pareto improvement on the conditional expected utilities of traders except h_0^* at s_1. This improvement is illusionary since the tax to finance the policy is ignored by the traders. Let's

Table 2: A comparison of expected utilities when $M \geq (1-R)\, q_{h_0^*}$

Trader Group	Security Holding w.o. policy→w. policy	E.U. at s_1 w. Policy	E.U. at s_1 w.o. Policy
$[0, \bar{h}_1]$	Bond→Bond	$\frac{1+p^*(1)}{h_0^*}$	$\frac{1+p(1)}{h_0^*}$
$(\bar{h}_1, h_0^*]$	Equity→Bond	$\frac{1+p^*(1)}{h_0^*}$	$\frac{q_h}{q_{\bar{h}_1}} \frac{1+p(1)}{h_0^*}$
$(h_0^*, h_1^*]$	Equity→Bond	$\frac{M}{1-h_0^*}$	0 (bankrupt)
$(h_1^*, 1]$	Equity→Equity	$\frac{q_h}{q_{h_1^*}} \frac{M}{1-h_0^*}$	0 (bankrupt)

consider admissible tax schemes at each state in the period 2 and check whether the traders can be still improved after tax or not. I assume, again, that the government actually does not intend to save those bankrupt at s_1 without the policy.

Note that, with the policy M^*, the government can identify three trader groups in the table by observing the asset holdings. The first group, $[0, h_0^*]$, consists of the bond/promise buyers at s_0. The second group $(h_0^*, h_1^*]$ consists of traders who buy the equity at s_0 but the bond/promise at s_1. The third group, $(h_1^*, 1]$, consisted of the equity buyers at s_1. In each group, traders have the same income at s_1, so they also have the same payoff (utility) at each state in the period 2. I call these groups the pool #1, #2, and #3, in the order I mentioned.

At s_2, traders in the pool #3 are bankrupt, so that only the pool #1 and #2 can be taxed. The maximum amount the government can levy from the pool #2 is $\frac{h_1^*-h_0^*}{1-h_0^*}M^*$, which is less than M^*. Since the government cannot distinguish a trader from others in the same pool, it must levy $\frac{1}{h_0^*}\times\frac{1-h_1^*}{1-h_0^*}M^*$ from each trader in the pool #1. This leaves a positive payoff to each trader in the pool #1, which is calculated as $\frac{1+R}{h_0^*}$. For a trader in $[0, \bar{h}_1]$, this payoff (utility) is worse than that without the policy, $\frac{1+p(1)}{h_0^*}$, since $p(1)>R$. For a trader h in $(\bar{h}_1, h_0^*)$, it is clearly better than that without the policy, 0. As a result, those buying the bond/

promise regardless of the policy at s_1 are victimized by the policy, while those buying the equity without the policy at s_1 are improved after tax.[5]

At s_3, the total payoff that traders in the pool #2 and #3 receive is $\frac{1-R}{p^*(1)-R}\frac{1-h_1^*}{1-h_0^*}M^*+\frac{h_1^*-h_0^*}{1-h_0^*}M^*$. Since $p^*(1)<1$, this total payoff exceeds M^*. Hence the government can finance M^* only by taxing on the pool #2 and #3, leaving a positive payoff for all of them. (For example, since the difference between the total payoff on pool #2 and #3 combined and M^* is $\Delta\equiv\frac{1-p^*(1)}{p^*(1)-R}\times\frac{1-h_1^*}{1-h_0^*}M^*>0$, the government can leave $\frac{\Delta}{1-h_0^*}$ to each trader in the pool #2 and pool #3, and take away all the rest by tax.) Both would be bankrupt if the policy was not implemented, so the economic welfare on them is improved. In the pool #1, the payoff for each trader in $[0, \bar{h}_1]$ is increased from $\frac{1+p(1)}{h^*}$ to $\frac{1+p^*(1)}{h^*}$. However, for each trader in $(\bar{h}_1, h_0^*]$ the payoff must be decreased, since the total payoff after tax available at s_3 is unchanged from that without the policy (equal to 2), while all traders in other groups are better off. The government may wish to save those in $(\bar{h}_1, h_0^*]$, but it cannot since, having no way of knowing $\bar{h}_1$, it cannot distinguish them from others in the pool #1. To

5 It is possible for the government to leave a small positive payoff to those in pool #3 and still be feasible to finance M^*. In this case, each trader in the pool #1 is taxed more, so that the bond/promise buyers regardless of the policy are hurt even more.

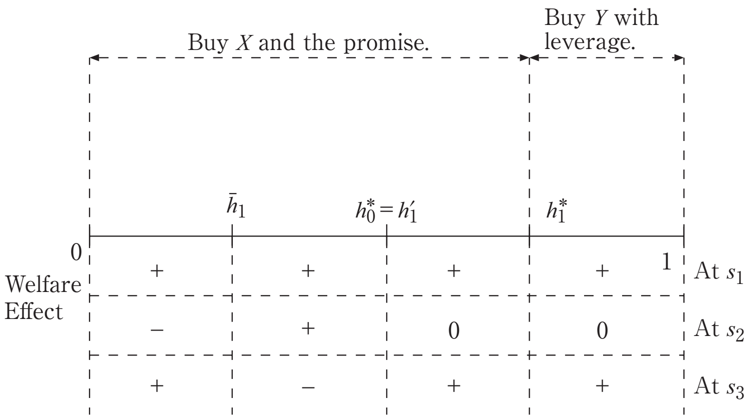
save them, it must reward all traders in the pool #1, which results in increasing the already increased payoffs of those in $[0, \bar{h}_1]$. With the total payoff after tax unchanged from that without the policy, the resource is never enough to improve the intended target, even if the government takes away entire payoff from the pool #2 and #3. Hence, at s_3, those who would have been bankrupt and those who would have been buying the bond/promise at s_1 are better off, but those who would have been buying the equity at s_1 are worse off.

Over all, in period 2, those would have been buying the equity without the policy are victimized if the state is "good", while those who would have been buying the bond/promise without the policy are victimized if the state is "bad". The analysis is summarized in Figure 3.

The result is summarized in the following proposition.

Proposition 6.1 *Consider the model described, and assume that the govern-*

Figure 3: Welfare effect of the large scale policy



ment implements a temporal credit–saving policy of the type 1, M, at s_1. If $M < (1-R)q_{h_0^*}$, then traders in $(h'_1, h_0^*]$ are strictly worse off at s_1, those in $[0, \bar{h}_1]$ are strictly worse off after tax at s_2, and those in $(\bar{h}_1, h_1^*]$ are strictly worse off after tax at s_3, where h'_1 is the trader who achieves the same conditional expected utility at s_1 by buying the equity with leverage without the policy and by buying the bond/promise with the policy. Otherwise, the government implements the policy M^* for which $h'_1 = h_0^*$. With this policy, all traders are better off at s_1, but, at s_2, traders in $[0, \bar{h}_1]$ are strictly worse off after tax, while traders in $(\bar{h}_1, h_0^*]$ are better off after tax. At s_3, traders in $(\bar{h}_1, h_0^*]$ are strictly worse off after tax, while all other traders are strictly better off after tax.

7 Evaluation of the Policy and a Satisfactory Action

As explained in the section 4, the policies are evaluated in two criteria:

1. Do they achieve a Pareto improvement of the allocation when they are active?
2. Do they achieve a Pareto optimal allocation when they are active?

Since the assurance of Pareto optimality requires the completeness of the financial market in this model, and the temporal credit-saving policy does not affect the span of the financial market, it fails on the second criterion. The argument used in the end of section 3 can be applied to show that the equilibrium allocation with the policy is Pareto suboptimal at s_1.

For the type 1 policy M^*, a Pareto improving allocation is obtained as follows. For traders in $(h_1^*, 1]$, allocate the same amount of the Arrow security on s_3 as that in the equilibrium. This uses up $(1-R)$ out of the total $(2+M)$ Arrow security on s_3, and leaves $(1+R+M)$ of both Arrow securities to be allocated to traders in $[0, h_1^*]$. Using them, create $(1+R+M)$ of the bond and distribute to traders in $[0, h_1^*]$ so that each trader receives exactly the same income as that in the equilibrium, if the price of the bond is normalized to 1. Then, obtain an equilibrium allocation of the economy in which the group $[0, h_1^*]$ trades two Arrow securities. This equilibrium allocation improves the allocation in the original equilibrium for this group.

The proposition 6.1 says that it also fails on the first criterion. Hence the temporal credit-saving policy is not worth implementing, though the type 1 policy M^* achieves an illusionary Pareto improvement at the state the policy is implemented.

Then, what action should be implemented？ On the second criterion, the answer is already given in Fostel and Geanakoplos (2011). It is enough to admit tranching on both the bond and the equity. As they explain, this creates the Arrow security on the "bad" state in the equilibrium, which can be sold by both bond holders and equity holders. The left-over after the tranching is the Arrow security on the "good " state, for both assets. Hence both bond holders and equity holders can buy and sell all Arrow securities, making the financial market complete.

The timing to introduce tranching is important on the first criterion. The tranching on both the bond and the equity must be introduced before trading occurs at s_0.[6] Since the action does not do anything after the period 1, its welfare effect must be evaluated only in terms of traders' expected utilities at s_0. Since the trading in this model has not started yet there, it has no alternative case to be compared with. Therefore it achieves both a Pareto improvement and a Pareto optimal allocation.[7] The appendix B in Kaneko (2011) analyzes what happens if the tranching on the bond and the equity is introduced at s_1, and proves that there exists a group of traders whose expected utilities at s_1 are worse off by such an action. This analysis becomes possible since the trading in this model has already started at s_0, so that we have a hypothetical equilibrium allocation to be compared with at s_1. There exists a trader $h_1^* < \bar{h}_1$, who is indifferent between holding the Arrow security on s_2 and holding that on s_3. All traders in $[0, h_1^*)$ buy the Arrow security on s_2, while all traders in $(h_1^*, h_0^*]$ buy the Arrow security on s_3. Traders sufficiently close to h_1^* are worse off, while those far from h_1^* can be better off. Especially, those in $[\bar{h}_1, h_0^*]$ are better off since they are optimistic enough to bet on s_3. There are victims in both group of traders, and they are "irresolute" when h_1^* is sufficiently close to $\bar{h}_1$.[8] But, when h_1^* is small, all buyers of the Arrow security on s_2 are worse off.

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6 For the importance of timing to introduce tranche and/or CDS on a price dynamics and a credit cycle, see Fostel and Geanakoplos (2011).

7 Surviving the criteria I propose does not necessarily means that the realized allocation looks reasonably moderate in a common sense. In this model, it is quite opposite in the sense that we will see some bankrupt traders in every state after the period 1, even at "good" states. This is an unwanted consequence of the assumption that all traders are risk-neutral. In the complete market, all risk-neutral traders bet on the states which they give the highest subjective probability. Therefore the casualty of bankruptcy is maximized in the complete market. Replacing risk-neutral traders by risk-averse traders may improve on that matter, but the price to pay is that the calculation of the equilibrium allocation will be made far more difficult.

8 Those "irresolute" are victimized since they are invited into betting on a state even if they are not so sure which state is likely to occur.

In reality, all economic agents exist in the state at which trading in the economy has started a long time ago. Hence the analysis in the appendix B of Kaneko（2011）would give some insight about what happens if the tranching on all securities is introduced at some point in the real world. Especially it would imply that, if the authority who can take an action does not have an ability to distinguish one agent from another, there will be no fully satisfying action in terms of my criteria in the real world, and only the Pareto optimality of an equilibrium allocation will be achieved by the introduction of tranching on all securities.

The analysis given is summarized in the following theorem.

Theorem 7.1 *Consider the model described. The temporal credit-saving policy of type 1 fails on both of two criteria proposed. Introducing tranching on both the bond and the equity at* s_0 *before a trading occurs satisfies both criteria. If it is introduced at* s_1*, it fails on the first criterion but satisfies the second criterion.*

Some readers may think that, if the introduction of tranching at s_1 is reinforced by a proper policy by the government, it may bring a Pareto improvement. The appendix C in Kaneko（2011）analyzes the welfare effect of such an action, where the government transfers income from the bond/promise holders at s_1 to the equity holders at s_1. It shows that, if a condition is met, this reinforcement achieves a Pareto improvement. However, the existence of the income transfer for which this condition is satisfied depends on the function q. Even when it exists, the government lacks an ability to calculate it since it has no way of knowing q.

8 Conclusion and Remarks for Further Study

The results in this article suggest strongly that the temporal credit-saving policy of type 1 is not worth taking. However, it might be if two factors, which are left out of the scope in this article, are taken into account. One is a positive real effect of the policy. The other is a liquidity.

A positive real effect of a monetary policy, with a possible support by an adequate fiscal policy, is believed to exist in some area of macroeconomics. In the model of this article, it is embodied as an increase in the payoff of the equity, since this payoff comes from the production in the real sector of the economy. In the appendix A, I give an analysis of the welfare effect of the type 1 policy that has a positive real effect. There, it is assumed that the type 1 policy M increases the payoff at s_2 by $\delta_M > 0$. The real effect δ is assumed to be an increasing continuous function of M, though the increased payoff $(R+\delta_M)$ is always less than 1, the payoff when the economy turns out to be "good". The analysis shows that, by sacrificing those who hold the equity regardless of the policy at s_3, (1) the type 1 policy in a small scale makes all traders better off in s_1 and s_2 if the real effect of the policy is strong and q is moderately responsive to h positively around $\bar{h}_1$, and (2) that in a large scale makes all traders better off at s_1 and s_2 if the real effect can be made very strong. The efficacy of a small scale policy depends on a subtle relation between the strength of the real effect of the policy and the variety of trader types, so that it is hard to be achieved by the government. The efficacy of a large scale policy depends only on the strength of the real effect of the policy, so that the result may be seen as a theoretical evidence that a "massive" money injection, such as a quantity easing policy with an inflation target, improves the welfare of traders if a bad turn of the economy is highly expected in the real side of the market. However, this efficacy is a consequence of a strong real effect of money, that is rarely observed in empirical studies. Regarding the quantity easing policy in Japan, Chapter 10 in Umeda (2011) states that preliminary empirical researches in BOJ find no conclusive evidence on a positive real effect of the policy, and that has become a consensus among the members of the Policy Board at BOJ. Arai (2012) finds that the small stock market bubbled in the period that the quantity easing policy was prolonged, and such a price increase was highly correlated with the offshore money account, which is considered to capture the strength of traders' expectation that the quantity easing policy will stay. These indicate that it is highly unlikely that a strong positive real effect of a temporal credit-saving policy exists.

It is hard to analyze the problem of liquidity in the model adopted in this article, since it is a general equilibrium model so that all assets are traded without any shortage of liquidity at s_1. Note that BOJ now emphasizes that the positive aspect of the quantity-easing policy was to make the yield curve flatter by reducing the long-term interest rate. The long-term commitment on the quantity-easing policy until the rate of increase in the general price index becomes stabilized at 0% achieved this by creating a market expectation that there would be more than enough money supply in the future. Such an effect is called as a "time-axis effect". It is clear that this effect is a recovery of liquidity on the long-term bonds. The 3-period model used in this article has no way of distinguishing the long-rate from the short-rate, so that this effect cannot be analyzed at the outset. To analyze such a liquidity issue on the bonds with different maturities, either the time-horizon should be extended to incorporate bonds with longer terms or a bond with a short maturity within a period should be introduced, along with a device to incorporate a notion of liquidity.

Since a temporal credit-saving policy is implemented as a monetary policy in reality, a monetary economics model may be more appropriate if we want to focus on policy tools currently available for central banks. In Dubey and Geanakoplos (2001), a neo-classical static monetary economics model is proposed and the roles of a monetary policy (injection of inside-money) and a fiscal policy (injection of outside-money) are analyzed. The model is extended to 2 periods with uncertainty in Dubey and Geanakoplos (2003), but the analysis of policies are left unsolved. In general, a recovery of liquidity and a positive real effect are not independent. For the models in these papers, a positive real effect of a monetary policy can be brought only by easing liquidity constraints, since such an effect comes from an increase of consumption by easing the cash-advanced constraints of consumers through the reduction of a borrowing cost. However, the experience of Japan strongly suggests that the recovery of liquidity alone will not be enough to bring a recovery of the real economy. It was mentioned earlier that a time-axis effect of the quantity-easing policy had existed clearly, but no positive real effect had been detected. Without a doubt, an increase of consumers' income is crucial. In the case of Japan, it is conceived that a shortage of liquidity had existed only in long-term bonds, so that a positive effect on the consumption demand from a recovery of their liquidity was limited to durables and novel goods in the future which are expected to have high prices, and, as such, was entirely offset by a negative perspective of diminishing income for a long time. Also, the lowered long rate would have helped firms engaging in a resource-consuming production of durables and developing a new technology to produce novel goods, but the perspective of a shortage in the demand prevented it to happen. Thus, the lack of an adequate fiscal policy might have been a crucial mistake in the case of Japan. By extending the time horizon to at least 3 and introducing a production into the model of Dubey and Geanakoplos (2003), all of these may be explained.

The model has only two securities and risk neutral traders. It is too limited to represent the complexity of the real world economy. However, admitting all shortcomings of the analysis in this article, the results obtained still help deducing a rough idea about the consequence of a temporal credit-saving policy in reality.

For example, consider the problem of defaulting countries in EU. The bonds issued by those defaulting countries is the equity Y in the model. The bonds issued by non-defaulting countries is the bond X in the model. Buying out the bonds of defaulting countries by ECB may be viewed as a temporal credit-saving policy of type 1. Note that the action do not specify any financing plan in detail when it is implemented. The proposition 6.1 gives the following prediction.

(1) If this buyout is small in scale, the safe bond holders discouraged by higher prices of the bonds of defaulting countries are worse off at the state where this buyout is performed, since the opportunity cost from the lowered return on the risky bond is not fully compen-

sated by the increase of their income.

(2) If this buyout is sufficiently large in scale, all traders are better off at the state where this buyout is performed. However a group of the safe bond holders are worse off in the future period. If the defaulting countries actually defaults, all traders who would also have held the safe bond without the buyout are worse off by a heavy tax burden. If the defaulting countries do not default, the safe bond holders who are discouraged by the higher prices of the risky bond are worse off since they have lost an opportunity to enjoy a decent return from their successful investment.

If EU issues a unified EU bond, it is regarded as Y, and the U.S. treasury bond and/or the Japanese treasury bond are regarded as X. The buyout of the unified EU bond by IMF, an international rescue fund etc. may be viewed as a temporal credit-saving policy of type 1. It is now easy to draw a similar prediction. In this case, U.S. treasury bond holders worldwide and/or Japanese nationals will be victimized.

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Appendix

A　A positive real effect of the type 1 policy

Here I assume that the type 1 policy M has a positive real effect in the economy, so that it increases the payoff of the equity at s_2 by δ_M.

This δ is an increasing continuous function of M, $\delta_0=0$, and $\delta_M<1-R$ for all $M>0$. I also assume that the type 1 policy M has no real effect at s_3.

In the unique equilibrium, only the promise $(R+\delta_M)$ is traded. The marginal buyer of the equity, h_1^*, is determined by

$$q_{h_1}{}^*=\frac{p^*(1)-(R+\delta_M)}{1-(R+\delta_M)}.$$

All traders in $[0, h_1^*]$ buy the bond/promise, while all traders in $(h_1^*, 1]$ buy the equity with leverage $(R+\delta_M)$.

The unknowns of the equilibrium are the marginal buyer h_1^* and the equity price at s_1, $p^*(1)$. Depending on whether $h_1^*<h_0^*$ or not, they are determined by the following system of equations.

When $h_1^*<h_0^*$:

$$\begin{cases} p^*(1)=\dfrac{h_0^*}{h_1^*}(1+R+\delta_M+M)-1, \\ \\ q_{h_1^*}=\dfrac{1}{1-(R+\delta_M)}\left[\left(\dfrac{h_0^*-h_1^*}{h_1^*}\right)(1+R+\delta_M)+\dfrac{h_0^*}{h_1^*}M\right]. \end{cases} \tag{5}$$

When $h_1^*\geq h_0^*$:

$$\begin{cases} p^*(1)=\dfrac{1-h_1^*}{1-h_0^*}M+(R+\delta_M), \\ \\ q_{h_1^*}=\dfrac{1-h_1^*}{1-h_0^*}\dfrac{M}{1-(R+\delta_M)}. \end{cases} \tag{6}$$

In both cases, the r.h.s. of the equation for $q_{h_1^*}$ is increasing with respect to M and decreasing with respect to h_1^*. This implies that it has at most one solution h_1^* for each M, and h_1^* increases as M increases. Since $h_1^*=\bar{h}_1$ when $M=0$, this implies that $h_1^*>\bar{h}_1$ for all $M>0$. A simple calculation shows that $p^*(1)$ is also increasing with respect to M.

Here $p^*(1) > p(1)$ for $M > 0$.

From the system of equations, it can be easily shown that,

$$h_1^* \lesseqqgtr h_0^* \Leftrightarrow M + q_{h_0^*}\,\delta_M \lesseqqgtr q_{h_0^*}(1-R).$$

Since the equilibrium system of equation differs depending on whether $h_1^* < h_0^*$ or not, I analyze two cases separately.

<u>Case 1: $M + q_{h_0^*}\,\delta_M < q_{h_0^*}(1-R)$.</u>

In this case, we have $\bar{h}_1 < h_1^* < h_0^*$.

First. consider the welfare effect at s_1. As a comparison of expected utilities, we obtain exactly the same table as that with no positive real effect. Traders in $[0, \bar{h}_1]$ are better off since their income is increased due to the increase of the equity price. Traders in $(h_0^*, 1]$ are also better off since they avoid bankruptcy. For traders in $(h_1^*, h_0^*]$, a calculation shows that they are better off if and only if the following inequality is satisfied,

$$h_1^* > \left(1 + \frac{M}{1+R+\delta_M}\right)\left(\bar{h}_1 + (h_0^* - \bar{h}_1)\frac{\delta_M}{1-R}\right). \tag{7}$$

(It is calculated using $\left(\frac{1+p^*(1)}{h_0^*}\right)/q_{h_1^*} = (1-(R+\delta_M))\frac{1+R+\delta_M+M}{(h_0^*-h_1^*)(1+R+\delta_M)+h_0^* M}$, and $\left(\frac{1+p(1)}{h_0^*}\right)/q_{\bar{h}_1} = \frac{1-\mathrm{R}}{h_0^*-\bar{h}_1}$.)

Whether this inequality is satisfied or not depends on the properties of the functions q and δ. At least it requires that q is not very responsive to h compared with the response of δ to M in a relevant range. For those in $(\bar{h}_1, h_1^*]$, the expected utility with the policy is the same for all of them, and that without the policy increases with h. Hence, once the trader h_1^* is better off, then all of them are better off. The inequality (7) ensures that. I have shown that, if the inequality (7) holds, then all traders are better off by the policy at s_1, though the improvement is illusionary since the traders do not take the tax burden in the period 2 into account.

Next, consider the welfare effect at s_2. All traders in $(h_1^*, 1]$ are bankrupt. Traders in $(h_1^*, h_0^*]$ would be also bankrupt without the policy. Traders in $(h_0^*, 1]$ would be inactive without the policy. Hence these traders neither gain nor loose. After all securities pay out, the government levies $\frac{M}{h_1^*}$ from each trader in $[0, h_1^*]$. This leaves a positive amount of payoff to each trader in $[0, h_1^*]$, which is $\frac{1+R+\delta_M}{h_1^*}$. All traders in $(\bar{h}_1, h_1^*]$ are better off since this positive payoff is better than being bankrupt. For traders in $[0, \bar{h}_1]$, the necessary and sufficient condition for that their payoff is better than their payoff without policy, $\frac{1+R}{\bar{h}_1}$, is

$$h_1^* < \left(1+\frac{\delta_M}{1+R}\right)\bar{h}_1. \tag{8}$$

Though $h_1^* > \bar{h}_1$ implies that their payoff is depreciated more with the policy, the positive real effect of the policy δ_M may offset it. Whether this condition is met or not depends on the functions q and δ. It requires that the real positive effect δ_M is sufficiently large in the relevant range of M.

Finally, consider the welfare effect at s_3. Since the return on the equity exceeds 1 in this state, the government can finance the policy by levying $\frac{M}{1-h_0^*}$ from each trader in $(h_0^*, 1]$, leaving a positive payoff. Traders in $[0, \bar{h}_1]$ are better off, since they hold the bond/promise regardless of the policy, and $p^*(1) > p(1)$. All traders in $(h_1^*, h_0^*]$ hold the equity regardless of the policy, so that, by the same argument as that at the state s_1, they are better off if and only if the inequality (7) is satisfied. All traders in $(\bar{h}_1, h_1^*]$ receive an equal payoff $\frac{1+R+\delta_M+M}{h_1^*}$. They would be holding the equity without the policy, so they would receive $\frac{1}{q_{\bar{h}_1}} \frac{1+R}{\bar{h}_1}$. Without a calculation. we know that they must be worse off by the policy, since the total payoff at s_3 after tax, which is 2, is unchanged and all traders other than those in $(\bar{h}_1, h_1^*]$ are better off.

Over all, the policy may improve all traders at s_1 and s_2, sacrificing those who hold the equity regardless of the policy at s_3, if and only if the following condition is satisfied,

$$\left(1+\frac{M}{1+R+\delta_M}\right)\left(\bar{h}_1+(h_0^*-\bar{h}_1)\frac{\delta_M}{1-R}\right)<h_1^*<\left(1+\frac{\delta_M}{1+R}\right)\bar{h}_1. \tag{9}$$

The result looks encouraging at a glance, since it suggests a possibility that the type 1 policy in a small scale may improve all traders in states where the economy turns bad. However, we should not be optimistic. The condition requires that the positive real effect of the policy is sufficiently large on the specified range of M in this case, and q is moderately responsive to h positively around $\bar{h}_1$. Thus whether it is met or not is a subtle matter, since a strong positive real effect of the policy alone does not guarantee the improvement. Also, the government can never be sure about the improvement since it has no knowledge of q, and is also likely to lack a knowledge of δ.

Case 2: $M+q_{h_0^*}\delta_M \geq q_{h_0^*}(1-R)$.

In this case, $h_1^* \geq h_0^*$. The analysis is parallel to the one given in the section 6.

First, consider the welfare effect at s_1. Traders in $(h_0^*, 1]$ are better off, since they receive a positive income with the policy, which is better than being bankrupt. Traders in $[0, \bar{h}_1]$ are better off since they hold the bond/promise regardless of the policy and their income is increased. For those in $(\bar{h}_1, h_0^*]$, the argument proceeds just as that in the section 6. Let h'_1 be the trader who is indifferent between buying the equity without the policy and buying the bond with the policy. It is determined by $q_{h'_1}=(\frac{1+p^*(1)}{1+p(1)})\times q_{\bar{h}_1}$, so that $h'_1>\bar{h}_1$. If $h'_1<h_0^*$, the traders in $(h'_1, h_0^*]$ are worse off. Hence the government must choose M for which $h'_1 \geq h_0^*$. Let M be any level to achieve $h'_1 \geq h_0^*$. With this M, all traders in $(\bar{h}_1, h_0^*]$, possibly except h_0^*, are in the range less than h'_1, so that they are strictly better off. (The trader h_0^* may neither gain nor lose, but is not worse off.)

Next, consider the welfare effect at s_2. Traders in $(h_1^*, 1]$, who would be inactive without the policy, are bankrupt. Hence pools that the government can tax are $[0, h_0^*]$ and $(h_0^*, h_1^*]$, which I call the poll #1 and #2 in order. The total payoff of the pool #2 is less than M, since this groups holds the bond/promise whose return is 1. The government takes away all of that by tax, which amounts to $\left(\dfrac{h_1^* - h_0^*}{1 - h_0^*}\right)M$. Traders in the pool #2, who would be inactive without the policy, are therefore bankrupt. The government must collect the rest from the pool #1, and it levies $\dfrac{1}{h_0^*}\dfrac{1-h_1^*}{1-h_0^*}M$ from each trader in the pool #1. A simple calculation shows that each trader in this pool has the payoff after tax equal to $\dfrac{1+R+\delta_M}{h_0^*}$. Traders in $(\bar{h}_1, h_0^*]$ are better off since they would be bankrupt without the policy. Each trader in $[0, \bar{h}_1]$ would have $\dfrac{1+R}{\bar{h}_1}$ without the policy, and he is better off by the policy if and only if $\delta_M > \dfrac{h_0^* - \bar{h}_1}{\bar{h}_1}(1+R)$.

Finally, consider the welfare effect at s_3. Each trader in $(h_1^*, 1]$ receives $\dfrac{1}{q_{h_1^*}}\dfrac{M}{1-h_0^*}$, which is larger than $\dfrac{M}{1-h_0^*}$ since the return on the equity exceeds 1. Since each trader in $(h_0^*, h_1^*]$ receives $\dfrac{M}{1-h_0^*}$ from the bond/promise, those who would be bankrupt at s_1 without the policy has a total payoff more than M. Therefore the government can levy M from traders in $(h_0^*, 1]$ and still leave positive payoffs to all of them. Traders in $[0, h_0^*]$ are not taxed, and each receives $\dfrac{1}{h_0^*}\left[\dfrac{1-h_1^*}{1-h_0^*}M + (1+R+\delta_M)\right]$ from the bond/promise. Traders in $[0, \bar{h}_1]$ are better off since they hold the bond/promise regardless of the policy and their income at s_1 is increased by the policy. Since the total financial payoff after tax at s_3, which is 2, is not changed by the policy, all traders in $(\bar{h}_1, h_0^*]$ must be worse off. Since the payoff is equal for each trader in $[0, h_0^*]$, the policy of a larger scale may not reduce the welfare loss of these traders.

In summary, the policy may improve all traders at s_1 and s_2, sacrificing those who hold the equity regardless of the policy at s_3, if and only if the following inequality is satisfied,

$$\delta_M > \frac{h_0^* - \bar{h}_1}{\bar{h}_1}(1+R). \tag{10}$$

The inequality does not depend on h_1^*, so that it can be satisfied if there is a strong positive real effect at a "large" M for which $h_1' \geq h_0^*$. The result looks encouraging, but again we must be cautious. First, $\delta_M < 1 - R$ no matter large M is, so that it may be impossible to satisfy the inequality if its r.h.s. is more than or equal to $(1-R)$. Secondly, the government is never sure about the scale of the policy for the improvement at s_1 and s_2. There are two reasons for this. One is that $\bar{h}_1$ is made counter-factual by the policy, so that the government cannot know the value of the r.h.s. precisely. The other is that the government is likely to lack the knowledge of δ.

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Summary

An Evaluation of Temporal Credit-Saving Policies I
—Welfare Analysis in a Simple Financial Trading Model with Heterogeneous Risk-Neutral Traders—

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I show that a temporal credit-saving policy by which bankrupt traders obtain credits do not improve the economic welfare. The analysis is based on a 3-period general equilibrium model with risk-neutral traders, in which a bond and an equity are traded and an opportunity for leverage is available. The implementation of the policy shifts the equilibrium and the payment of the financial contract in the past is adjusted according to its content. This adjustment creates an illusion that the policy improve the welfare of traders since the tax to finance the policy is not foreseen. However, after the tax is levied in the final period, in any state, a group of bond holders is worse off. The resulting allocation is Pareto suboptimal.

Key words: temporal credit-saving policy, quantity-easing policy, bankruptcy, incomplete market, leverage.
JEL classification codes: D52, D53, D61, G28.